Characterizing nonclassical correlations via local quantum uncertainty

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This talk is about…

Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton’s laws is one of the unresolved problems of physics. Figure 1

Zurek Physics Today 1991
Entanglement: a resource for novel information processing

Quantum cryptography

Quantum teleportation

Quantum computation
Quantum correlations

• In certain schemes of quantum computation where the quantum bits are affected by noise, there seems to be a speed-up over classical scenarios even in the presence of negligibly small or vanishing entanglement (Knill-Laflamme PRL 1998; Datta et al PRL 2008; Barbieri et al PRL 2008)

• What is the ultimate resource making quantum processors more effective than classical ones?

• A more refined analysis of the nature of correlations in composite quantum systems is in order
Correlations

Classical correlations

Quantum correlations
Correlations

- Pure bipartite states:
  - \textit{entanglement} = nonlocality
    nonclassicality (quantum correlations)

- Mixed bipartite states:
  - \textit{Werner 1989}: separable classically correlated
Quantumness in separable states

- Nonorthogonal separable states cannot be discriminated exactly
- Measuring a local observable on a separable bipartite state can perturb the state
- The eigenvectors of a separable state can be entangled superpositions

... 

In general separable states *have not* a purely classical nature
A better paradigm

Definition 1. A bipartite state $\rho$ is: (i) separable \cite{20} if it can be written as $\sum_i p_k \sigma^A_k \otimes \sigma^B_k$, where $p_k$ is a probability distribution and each $\sigma^X_k$ is a quantum state, and entangled if non-separable; (ii) classical-quantum (CQ) if it can be written as $\sum_i p_i |i\rangle\langle i| \otimes \sigma^B_i$, where \{|i\rangle\} is an orthonormal set, \{p_i\} is a probability distribution and $\sigma^B_i$ are quantum states; (iii) classical-classical (CC), or (strictly) classically correlated \cite{3, 5}, if there are two orthonormal sets \{|i\rangle\} and \{|j\rangle\} such that $\rho = \sum_{ij} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$, with $p_{ij}$ a joint probability distribution for the indexes $(i, j)$.

Discordant states

- Almost all bipartite states have non-classical correlations (classical-quantum and strictly classically correlated states are of zero measure)

A. Ferraro et al. PRA 2010
Quantum computation

Non-classical correlations without entanglement might explain the computational speed-up in the DQC1 model of noisy quantum computation

A. Datta et al. 2008-2011; experiments: M. Barbieri et al. PRL 2008 (photons); G. Passante et al., PRA 2011 (NMR)
Quantumness in separable states

Nonorthogonal separable states cannot be discriminated exactly.

Measuring a local observable on a separable bipartite state will perturb the state.

The eigenvectors of a separable state can be entangled superpositions.

In general, separable states have not a purely classical nature.

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THE POWER OF DISCORD

BY ZEeya MERALI

Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along?
Outline

- Uncertainty on a Single Observable
- Quantum vs Classical Uncertainty
- Quantum vs Classical Correlations
- Local Q Uncertainty = Q Correlations
- Q Correlations-Assisted Metrology
Measurement uncertainty

- We have a quantum system in the state $\rho$
- We measure a single observable $K$
- Uncertainty = error bar
Measurement uncertainty

- We quantify the total uncertainty (error bar) by the **variance** $\text{Var}_\rho K$
- This may contain **classical ignorance**, due to the state mixedness, and a genuinely **quantum** part, arising due to noncommutativity between state and observable
We quantify the **quantum uncertainty** on a single observable via the Wigner-Yanase skew information.

\[ J(\rho, K) = -\frac{1}{2} \text{Tr} \left\{ \left[ \sqrt{\rho}, K \right]^2 \right\} \]
\[ \quad = \text{Tr}[\rho K^2 - \sqrt{\rho} K \sqrt{\rho} K] \]

*Wigner-Yanase PNAS 1963; Luo PRL 2003*
Quantum uncertainty

\[ J(\rho, K) = -\frac{1}{2} \text{Tr} \left\{ [\sqrt{\rho}, K]^2 \right\} = \text{Tr}[\rho K^2 - \sqrt{\rho} K \sqrt{\rho} K] \]

- \( J(\rho, K) = 0 \iff [\rho, K] = 0 \)
- \( J(\rho, K) = \text{Var}_\rho K \iff \rho \) is pure
- \( J(\rho, K) \) is convex
- many other properties…
Quantum uncertainty

Which quantum states unavoidably exhibit quantum uncertainty on the measurement of any single observable?

For global observables, none!
Quantum uncertainty

Which quantum states unavoidably exhibit quantum uncertainty on the measurement of any single local observable?

States with nonzero quantum discord!
Quantum uncertainty vs discord
Quantum uncertainty vs discord

Werner state
\[ \rho_{AB} = p |\phi^+\rangle\langle\phi^+| + \frac{1-p}{4} I_{AB}, \quad K = \sigma_{ZA} \otimes I_B \]
Local quantum uncertainty

- We focus on a general bipartite state $\rho \equiv \rho_{AB}$ and on local observables $K_A$ on subsystem $A$ (with nondegenerate spectrum).
- The minimum skew information on a single local observable defines the local quantum uncertainty (LQU):

$$\mathcal{U}_A(\rho) = \min_{K_A} \mathcal{I}(\rho, K_A)$$
Local quantum uncertainty

\[ \mathcal{U}_A (\rho) = \min_{K_A} I (\rho, K_A) \]

- The LQU is a \textit{bona fide} measure of discord
  - It is invariant under local unitaries and nonincreasing under quantum operations on \( B \)
  - It vanishes if and only if \( \rho \) is classical-quantum, \( \rho = \sum_i p_i |i \rangle \langle i|_A \otimes \tau_i B \)
  - It reduces to an entanglement monotone for pure states
Local quantum uncertainty

\[ \mathcal{U}_A (\rho) = \min_{K_A} I(\rho, K_A) \]

The LQU is a *computable* measure of discord for $2 \times d$ systems

\[ \mathcal{U}_A (\rho_{AB}) = 1 - \lambda_{\text{max}} (W_{AB}) \]

\[(W_{AB})_{ij} = \text{Tr} \left\{ \rho_{AB}^{1/2} (\sigma_i A \otimes \mathbb{I}_B) \rho_{AB}^{1/2} (\sigma_j A \otimes \mathbb{I}_B) \right\}, \]

with $i, j = x, y, z$.

- geometric interpretation
  (Hellinger distance)
Applications to metrology

- Parameter estimation with mixed probes

\[ U_\phi = e^{-i\phi H_A}, \text{ where } H_A \text{ is a nontrivial local Hamiltonian} \]
Applications to metrology

- Parameter estimation with mixed probes

$$\rho \xrightarrow{U_\phi} \rho_\phi \xrightarrow{\text{measurement}}$$

- For $\nu \gg 1$ repetitions of the experiment, the best estimator saturates the Cramer-Rao bound

$$\text{Var}(\tilde{\phi}_{best}) = \frac{1}{\nu F(\rho_\phi)}$$

- $F(\rho_\phi)$ quantum Fisher information
Applications to metrology

- Parameter estimation with mixed probes

\[ \mathcal{U}_A (\rho) \leq \mathcal{I}(\rho, H_A) = \mathcal{I}(\rho_\phi, H_A) \leq \frac{1}{4} \mathcal{F}(\rho_\phi) \]

for \( \nu \gg 1 \) repetitions and for the optimal estimator

\[ \text{Var}(\tilde{\phi}_{\text{best}}) \leq \frac{1}{4\nu \mathcal{U}_A(\rho)} \]
Applications to metrology

- Parameter estimation with mixed probes

- Discord measured by LQU sets a guaranteed upper bound on the variance of phase estimation with mixed probes and optimal measurements

\[
\text{Var}(\tilde{\phi}_{\text{best}}) \leq \frac{1}{4\nu U_A(\rho)}
\]
Conclusions

- Quantum states can display quantum uncertainty on a single observable, quantifiable via the Wigner-Yanase skew information.
- For a bipartite system, this is unavoidable as soon as there are discord-type nonclassical correlations.
- The minimum quantum uncertainty on a local observable defines a *bona fide* and computable measure of discord.
- Quantum discord in the mixed probe states used for phase estimation ensures that the optimal metrological scheme returns an estimator with variance bounded from above by the inverse of discord.

Thank you!