QUANTUM BENCHMARKS FOR GAUSSIAN STATES

Gerardo Adesso
School of Mathematical Sciences
The University of Nottingham

Giulio Chiribella
Center for Quantum Information
Tsinghua University, Beijing


http://quantumcorrelations.weebly.com
In order to implement quantum interfaces one needs to be able to:

✓ **Entangle** multiple nodes
✓ **Teleport** information through the channels
✓ **Store and retrieve** quantum states from light to matter
Quantum teleportation and storage

- are fundamental ingredients of quantum networks
- are implementations of the **identity** channel

- can be perfectly realised in principle by sharing maximally entangled states (assisted by classical communication)
Quantum teleportation and storage

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- are implementations of the identity channel
- are realised in practice with non-unit fidelity due to various sources of imperfections, noise, etc.
Quantum teleportation and storage

- are realised in practice with **non-unit fidelity** due to various sources of imperfections, noise, etc.

\[ |\psi\rangle_{in} \approx \mathbb{I} \]

\[ \rho_{out} \]

**Meme Image:**

*IF THE RESULT AIN'T GOING TO BE PERFECT...*

*DO WE REALLY NEED ENTANGLEMENT THEN?*
Quantum benchmarks

- set thresholds for the best approximation of the identity channel (or other channels) without quantum resources

Quality control for quantum technology
Teleportation

Fidelity

\[ F_Q = \langle \psi_{in} | \rho^Q_{out} | \psi_{in} \rangle \]
“Classical” strategy

Measure & Prepare

Fidelity

$F_{M&P} = \langle \psi_{in} | \rho_{out}^{M&P} | \psi_{in} \rangle$
Formalising the problem

- The input states are drawn from an input set $\Lambda = \{|\psi_{in}\rangle, p_\psi\}$ with a certain prior probability distribution $p_\psi$.
- The specifics of the input set are known to Alice and Bob.

- Average fidelities $\overline{F}$ are calculated (for classical or quantum strategies, respectively) by averaging over the input set

$$\overline{F} = \sum_{\psi \in \Lambda} p_\psi \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$$
Classical fidelity threshold (CFT)

- Best possible average fidelity without entanglement

\[
\bar{F}_C = \sup_{M \& P} \bar{F}_{M \& P} = \sup_{M \& P} \left( \sum_{\psi \in \Lambda} p_\psi \langle \psi_{in} | \rho_{out}^{M \& P} | \psi_{in} \rangle \right)
\]

- The CFT sets a **Quantum Benchmark** for transmission of states drawn from the set \( \Lambda \)

- Quantum implementations need to achieve \( \bar{F}_Q > \bar{F}_C \) to certify that entanglement has been used
Quantum benchmarks: known results

Input: **qudit states**
randomly sampled from $\mathbb{C}^d$

Benchmark: $\overline{F}_C = \frac{2}{d+1}$

*(Bruss-Macchiavello, PLA 1999)*

- Continuous variable limit: $\overline{F}_C \xrightarrow{d \to \infty} 0$

- Realistic experiments deal with a restricted class of states
  - Typically, **Gaussian states**
Gaussian states

- Very natural: ground and thermal states of all physical systems in the harmonic approximation regime
- Relevant *theoretical* testbeds for the study of structural properties of entanglement and correlations, thanks to the symplectic formalism
- Preferred resources for *experimental* unconditional implementations of continuous variable protocols
- Crucial role and remarkable control in quantum optics
  - coherent states
  - squeezed states
  - thermal states
Gaussian states teleportation/storage

- Quantum teleportation between light and matter (Nature 2006)
Pure single-mode Gaussian states

\[ |\psi\rangle = D(\alpha)S(\xi)|0\rangle \]

- Displacement operator: \( D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \)
- Squeezing operator: \( S(\xi \equiv s e^{i\theta}) = \exp \left( \frac{1}{2} \xi a^\dagger^2 - \xi^* a^2 \right) \)

\[
s = \frac{1}{2} \ln \left| \frac{l_1}{l_2} \right|
\]

\((\sqrt{2} \text{ Re } \alpha, \sqrt{2} \text{ Im } \alpha)\)
Input: **coherent states** \((\xi = 0)\)
with Gaussian displacement distribution, inverse width \(\lambda\)

**Benchmark:**

\[
\bar{F}_C^{coh} (\lambda) = \frac{1 + \lambda}{2 + \lambda} \xrightarrow{\lambda \to 0} \frac{1}{2}
\]

\(p_\lambda (\alpha) = \frac{\lambda}{\pi} \exp(-\lambda |\alpha|^2)\)

(first experiment: Furusawa et al. Science 1998)
Gaussian benchmarks: special cases

Input: **squeezed states** (unknown $s$; $\alpha, \theta = 0$) along one direction, with totally random squeezing degree

Benchmark: $\overline{F}_C \approx 0.83$

(Adesso-Chiribella PRL 2008)

Input: **squeezed states** (known $s$; unknown $\alpha, \theta$) with known squeezed degree, random phase and displacement

Benchmark: $\overline{F}_C = \frac{\text{sech} \ s}{2}$

(Owari et al NJP 2008)
(Calsamiglia et al PRA 2009)
(experiment: Jensen et al Nat Phys 2011)
**Gaussian benchmarks: this work**

**Input:** **squeezed states** (unknown \( s, \theta; \alpha = 0 \))
no displacement, random phase, squeezing degree distributed
according to a finite-width distribution \( p_{\beta} \) with inverse width \( \beta \)

**Benchmark:**
\[
\overline{F}^{sqz}_C = \frac{1 + \beta}{2 + \beta}
\]
(Chiribella-Adesso PRL 2014)

**Input:** **general Gaussian states** (unknown \( s, \theta, \alpha \))
with random phase, displacement and squeezing degree distributed
according to a finite-width distribution \( p_{\lambda,\beta} \) with inverse widths \( \lambda \) and \( \beta \)

**Benchmark:**
\[
\overline{F}^{gen}_C = \left(\frac{1 + \lambda}{2 + \lambda}\right) \left(\frac{1 + \beta}{2 + \beta}\right)
\]
(Chiribella-Adesso PRL 2014)
Technicalities

The benchmarks are generally probabilistic

- Alice measures the input $|\psi_x\rangle$ by the POVM $\{\hat{P}_y\}$, Bob reprepares $\hat{\rho}_y$
- They discard runs where the outcomes $y$ are outside a “success” set

$$\tilde{F}_c = \sum_{x \in X} \sum_{y \in Y_{suc}} p(x|suc) \frac{\langle \psi_x | \hat{P}_y | \psi_x \rangle}{\sum_{y' \in Y_{suc}} \langle \psi_x | \hat{P}_{y'} | \psi_x \rangle} \frac{\langle \psi_x | \hat{\rho}_y | \psi_x \rangle}{\langle \psi_x | \hat{\rho}_y | \psi_x \rangle}$$
Technicalities

- The proof uses group theory arguments. We derived a formalism to calculate benchmarks for \textbf{generalised} (Perelomov) \textbf{coherent states}
- The input distributions are generally non-Gaussian, but they originate from the overlap of the considered states with the vacuum, \( p \sim |\langle 0 | \psi \rangle| \)

\[
p^C_{\lambda}(\alpha) = \frac{\lambda}{\pi} e^{-\lambda |\alpha|^2}
\]

\[
p^S_\beta(\xi) = \frac{p^\beta(s)}{2\pi}, \quad \text{with} \quad p^\beta(s) = \frac{\beta \sinh s}{(\cosh s)^{\beta+1}}
\]

\[
p^G_{\lambda, \beta}(\alpha, s, \theta) = \frac{\lambda \beta}{2\pi^2} \frac{e^{-\lambda |\alpha|^2} \Re(e^{-i\theta} \alpha^2) \tanh s \sinh s}{(\cosh s)^{\beta+2}}
\]
Beating the benchmarks?

• No experiment yet has teleported arbitrary Gaussian states sampled from a finite-width distribution

• Continuous variable teleportation protocol (Braunstein-Kimble PRL 1998)
  • Shared entangled resource: a two-mode squeezed state with squeezing $r$
  • Alice performs a double homodyne, Bob displaces his mode according to the communicated results
  • The protocol is deterministic
Beating the benchmarks?

- No experiment yet has teleported arbitrary Gaussian states sampled from a finite-width distribution

- Hybrid teleportation protocol (Andersen-Ralph PRL 2013)
  - Shared entangled resource: a product of $N$ two-qubit maximally entangled states
  - Perform $N$ discrete-variable teleportation in parallel on a splitted input, then recombine the output at the end
  - This protocol is probabilistic
Comparison

• Hybrid protocol with \( N = 2 \) qubit channels vs
• Braunstein-Kimble with \( r = \text{arcsinh}(1) \) [i.e. \( \sim 3.8 \text{ dB} \)]
• In both cases the shared resources have equal entanglement and mean energy.
Comparison

- Hybrid protocol with $N = 2$ qubit channels vs
- Braunstein-Kimble with $r = \text{arcsinh}(1)$ [i.e. $\sim 3.8$ dB]
- In both cases the shared resources have equal entanglement and mean energy

- Teleportation of undisplaced squeezed states distributed according to $p_\beta$

Similar results for other cases…

(I. Kogias, S. Ragy, G.A. PRA 89 (2014))
Summary


• We obtained **quantum benchmarks** for the teleportation and storage of realistic ensembles of pure single-mode Gaussian states, including the general case

• Summary of values for the classical fidelity threshold:

\[
\begin{align*}
\text{Displaced, unsqueezed} & : \quad \frac{1 + \lambda}{2 + \lambda} \\
\text{Undisplaced, squeezed} & : \quad \frac{1 + \beta}{2 + \beta} \\
\text{Displaced, squeezed} & : \quad \left(\frac{1 + \lambda}{2 + \lambda}\right) \left(\frac{1 + \beta}{2 + \beta}\right)
\end{align*}
\]

(Hammerer et al 2005)
Outlook & Work in progress

- Derive benchmarks for other channels applied to Gaussian states (cloning, amplification, …)
- Derive benchmarks for more general classes of states
- Devise optimal protocols suitable to transmit squeezing
- Demonstrate an experimental violation of the benchmarks!

\[
\frac{1 + \lambda}{2 + \lambda} \quad \frac{1 + \beta}{2 + \beta} \quad \left(\frac{1 + \lambda}{2 + \lambda}\right)\left(\frac{1 + \beta}{2 + \beta}\right)
\]

(Hammerer et al 2005)
THANK YOU

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