ACTIVATING ENTANGLEMENT FROM QUANTUM DISCORD

GERARDO ADESSO

CEWQO 2014
CORRELATIONS

Classical correlations

Quantum correlations
CORRELATIONS

- Pure bipartite states:
  - entanglement = nonlocality
    nonclassicality (*one kind of quantum correlations*)

- Mixed bipartite states:
  - A hierarchy of different quantum correlations...
MANY SHADES OF QUANTUMNESS

- Classical
- Discordant
- Entangled
- Steerable
- Nonlocal

Partially device-independent quantum information processing

Fully device-independent quantum information processing

Teleportation, dense coding, ...

Classical
(TWO) SHADES OF QUANTUMNESS

**entangled**

(with respect to subsystem $A$)

\[
\rho_{AB} \neq \sum_k p_k \tau^k_A \otimes \nu^k_B
\]

the state is not **separable**

i.e. it cannot be created by LOCC

**discordant**

\[
\rho_{AB} \neq \sum_k p_k \tau^k_A \otimes |k\rangle\langle k|_B
\]

the state is not **quantum-classical**

i.e. it is not invariant under any local measurement on party B
A LOCAL MEASUREMENT...

Given a state $\rho_{AB}$, entangled or separable, and given a projective measurement on subsystem B, $\{\Pi^k_B\}$, with $\Pi^k_B = |k\rangle\langle k|_A$, then

$$\rho_{AB} \rightarrow \frac{\sum_k (I_A \otimes \Pi^k_B) \rho_{AB} (I_A \otimes \Pi^k_B)}{\text{tr}\left[\sum_k (I_A \otimes \Pi^k_B) \rho_{AB} (I_A \otimes \Pi^k_B)\right]}$$

$$= \frac{\sum_k (I_A \otimes |k\rangle\langle k|_B) \rho_{AB} (I_A \otimes |k\rangle\langle k|_B)}{\text{tr}\left[\sum_k (I_A \otimes |k\rangle\langle k|_B) \rho_{AB} (I_A \otimes |k\rangle\langle k|_B)\right]}$$

$$= \sum_k \frac{\langle k|_B \rho_{AB} |k\rangle_B}{\text{tr}[...]} \otimes |k\rangle\langle k|_B$$

$$= \sum_k p_k \tau^k_A \otimes |k\rangle\langle k|_B$$

...PROJECTS ANY STATE INTO A QUANTUM-CLASSICAL ONE
Quantum-classical (or classically correlated) with quantum discord.

If there is at least one local measurement I can perform without affecting my state

otherwise

EXAMPLES

- $\frac{1}{2} (|00\rangle + |11\rangle)(\langle 00| + \langle 11|)$ is entangled

- $\frac{1}{2} (|00\rangle\langle 00| + |1 + \rangle\langle 1 +|)$ is separable but with discord (with respect to B)

- $\frac{1}{2} (|00\rangle\langle 00| + |+ 1\rangle\langle + 1|)$ is quantum-classical

- $\frac{1}{2} (|00\rangle\langle 00| + |11\rangle\langle 11|)$ is fully classically correlated
HOW TO QUANTIFY QUANTUM CORRELATIONS
1. GEOMETRIC APPROACH

- Distance-based discord $D_d(\rho) = \inf_{\chi \in QC} d(\rho, \chi)$

- Examples of distance $d$ (has to be contractive):
  - Trace distance
  - Bures distance
  - Hellinger distance
  - Relative entropy
  - ...

Modi et al. PRL 2010
2. INFORMATIONAL APPROACH

\[ D_I(\rho) = \inf_{\Pi_B} [I(\rho) - I(\Pi_B\rho\Pi_B)] \]

- \( I = \) mutual information \( \rightarrow \) Zurek’s measure of discord: quantifies the correlations destroyed by minimally disturbing local measurements

- \( I = - (\) von Neumann entropy \( \rightarrow \) relative entropy of quantumness: quantifies the added noise due to a minimally disturbing local measurement (= Type 1 distance-based discord with relative entropy)

Zurek 2001; Horodecki’s 2002
3. "ACTIVATION" APPROACH

- Let us model a local measurement (Von Neumann)
3. “ACTIVATION” APPROACH

**Theorem.** The output premeasurement state $\tilde{\rho}_{AB:M}$ is entangled for all choices of the $U_B$ IF AND ONLY IF the initial system state $\rho_{AB}$ has nonzero discord (with respect to B)

$\rho_{A:B}$

$|0\rangle_M$

$U_B$

$\tilde{\rho}_{AB:M}$

Measurement disturbance

Entanglement with the apparatus

Piani et al. PRL 2011
Streltsov et al. PRL 2011
Piani & Adesso PRA(R) 2012
3. “ACTIVATION” APPROACH

- Activ. Discord ($\rho$) : $Q_E(\rho_{AB}) = \min_{U_B} E(\tilde{\rho}_{AB:M})$

- The minimum entanglement $E$ between the system AB and the apparatus M generated during a local (pre)measurement on subsystem B quantifies the initial discord in the system.

$\rho_{A:B}$ $U_B$ $|0\rangle_M$ $\tilde{\rho}_{AB:M}$
3. EXAMPLES

<table>
<thead>
<tr>
<th>Entanglement $E$</th>
<th>Quantum correlations $Q_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative entropy of entanglement, distillable entanglement</td>
<td>-Relative entropy of quantumness (alternatively defined geometrically or informationally)</td>
</tr>
<tr>
<td>Negativity</td>
<td>-Negativity of quantumness (equal to the <em>trace-distance discord</em> if B is a qubit)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

The correspondence is **hierarchical**

$$Q_E(\rho_{AB}) \geq E(\rho_{AB}) \text{ for every } E$$

“quantum correlations go beyond entanglement”
3. CLASSIFICATION

Piani and GA, PRA(R) 2012

**Initial state of the system $S=(AB)$**

- $\rho_{AB}$ is classically correlated (quantum-classical)
  - There is a choice of the measurement basis so that no entanglement is necessarily created between $(AB)$ and $M$

- $\rho_{AB}$ is separable but with nonclassical correlations (discord)
  - Always *bipartite entanglement* across $(AB):M$. The minimum over all measurement bases defines the initial discord of $AB$

- $\rho_{AB}$ is discordant and also initially entangled
  - Always *genuine tripartite* entanglement generated among $A,B,M$ in the premeasurement state

**Entanglement in the premeasurement state**

- $\rho_{AB}$ is classically correlated (quantum-classical)
- $\rho_{AB}$ is separable but with nonclassical correlations (discord)
- $\rho_{AB}$ is discordant and also initially entangled
EXPERIMENTAL SETUP

Adesso et al. PRL 2014
I. GENERATION

Input states of the system

\[ \rho_{AB}^{(q)} = q|\Psi^+\rangle\langle\Psi^+|_{AB} + \frac{1-q}{2} \left(|\Phi^+\rangle\langle\Phi^+|_{AB} + |\Psi^-\rangle\langle\Psi^-|_{AB} \right) \]

- \( q = 0 \): the states are classically correlated
- \( q > 0 \): the states are discordant (discord increases with \( q \))
- \( q > 1/2 \): the states are also entangled

A: polarization of photon 1
B: polarization of photon 2
M: path of photon 2

Adesso et al PRL 2014
II. ACTIVATION

• How to demonstrate an “iff”?
• How to sample a continuous space?

We define a “net” of discrete measurement settings for $U_B$, and derive lower bounds on the output AB:M entanglement (negativity) from the finite net of experimental data.

Adesso et al PRL 2014
III. CHARACTERIZATION

- We perform full state tomography of the three-qubit, two-photon premeasurement state at the output, for all chosen settings (28 bases for $U_B$ times 6 choices of $q$)

Fidelities above 90% in all settings

Adesso et al PRL 2014
RESULTS: VERIFYING THE “IFF”

Entanglement (negativity) between AB and M in the pre-measurement state, by full state tomography of the output

Adesso et al PRL 2014
The minimum negativity in the output premeasurement state, minimized over the $U_B$'s, is verified to coincide with the trace-distance discord in the input state, calculated numerically by a priori tomography before the activation step.

The output state of $A,B,M$, is found to possess genuine tripartite entanglement whenever the input state is not only discordant, but entangled as well. This is revealed by witness operators.

Adesso et al PRL 2014
ACTIVATING ENTANGLEMENT FROM QUANTUM DISCORD

SUMMARY
WHAT DID WE LEARN?

- Quantum correlations beyond entanglement (=discord) are present in almost all quantum states.
- They manifest in particular in the necessary state disturbance due to any local measurement on a subsystem.
- They can be mapped or “activated” into more familiar entanglement via a protocol that realizes a local premeasurement. Here experimentally demonstrated.
- Operationally, discord=minimum local coherence. For all tasks requiring coherence in a preferred basis, bipartite discord guarantees success even without knowing that basis.
- Applications: metrology, cryptography, entanglement generation & distribution, non-universal quantum computation, restricted communication and more to be found!
EXTENSION: CONTINUOUS VARIABLES?

NO-GAUSSIAN-ACTIVATION Theorem

Mista et al arXiv 2014

The premeasurement state remains separable!
EXTENSION: CONTINUOUS VARIABLES?

NON-GAUSSIAN-ACTIVATION Theorem

Gaussian state (separable but discordant)

NON-Gaussian operation

CNOT: $|m, n\rangle \rightarrow |m, m + n\rangle$

The IFF can be proven again. Entanglement is always activated from Gaussian discord.