OPTIMAL PERFORMANCE OF QUANTUM REFRIGERATORS

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Autonomous machines that cool by absorbing heat with no power source

Used on caravans or in rural areas where main electricity line is missing

However quite inefficient compared to conventional compression fridges

How to understand and possibly improve their optimal performance?

We need to model elementary (quantum) instances of these devices
**THE TRICYCLE**

- Prototype of any generic continuous thermal machine
- Includes absorption and power-driven refrigerators \((T_w \to \infty)\), heat engines and heat converters
- Three reservoirs: \(T_w > T_h > T_c\)
- Heat currents: \(\dot{Q}_\alpha\) \((\alpha = w, h, c)\)

**Thermodynamics 101**

1. \(\sum_\alpha \dot{Q}_\alpha = 0 \) \([1^{\text{st}} \text{law}]\)
2. \(\sum_\alpha \frac{\dot{Q}_\alpha}{T_\alpha} = -\dot{S} \leq 0 \) \([2^{\text{nd}} \text{law}]\)

Selective coupling to the baths via filtered frequencies $\omega_\alpha$

In absence of friction, heat leaks, etc.: single stationary rate $J$

Heat currents: $\dot{Q}_\alpha = \pm \omega_\alpha J$

**Thermodynamics 101**

1. $\sum_\alpha \dot{Q}_\alpha = 0$ \hspace{1cm} [1st law]
2. $\sum_\alpha \frac{\dot{Q}_\alpha}{T_\alpha} = -\dot{S} \leq 0$ \hspace{1cm} [2nd law]

Resonance: $\omega_w = \omega_h - \omega_c$

QUANTUM ABSORPTION FRIDGE

- $T_w > T_h > T_c$; $\omega_w = \omega_h - \omega_c$
- Cooling window:
  \[ \omega_c \leq \omega_c^{\text{max}} = \frac{(T_w-T_h)T_c}{(T_w-T_c)T_h} \omega_h \]
- Cooling power: $\dot{Q}_c$
- Coefficient of performance (COP):
  \[ \varepsilon = \frac{\dot{Q}_c}{\dot{Q}_w} \leq \varepsilon_C \]
- Carnot COP:
  \[ \varepsilon_C = \frac{1-T_h}{T_w} - 1 \]
- For reversible machines, $\varepsilon \rightarrow \varepsilon_C$
  at vanishing cooling power

One qutrit (3-level maser)

\[ T_w > T_h > T_c; \omega_w = \omega_h - \omega_c \]

Flat spectral densities

\[
\hat{H}^{\text{int}} = \sqrt{\gamma} (|0\rangle\langle 1| + |1\rangle\langle 0|) \alpha \otimes \hat{B}_\alpha
\]

\[
\hat{B}_\alpha = \sum_\mu k_{\alpha,\mu} \sqrt{\omega}_\mu (\hat{b}_{\alpha,\mu} + \hat{b}_{\alpha,\mu}^\dagger)
\]

- Palao, Kosloff & Gordon, Phys. Rev. E 64 (2001)
Two qubits

$T_w > T_h > T_c$; $\omega_w = \omega_h - \omega_c$

Flat spectral densities

$\mathcal{H}_\alpha = \frac{1}{\sqrt{\gamma}} (|0\rangle\langle 1| + |1\rangle\langle 0|) \otimes \hat{B}_\alpha$

$\mathcal{H}_w = \sqrt{\gamma} (|0\rangle_c \langle 1|_h + |1\rangle_c \langle 0|_h ) \otimes \hat{B}_w$

Three qubits

$T_w > T_h > T_c$; $\omega_w = \omega_h - \omega_c$

Flat spectral densities

Flat spectral densities


Models (1) and (2) are ideal reversible devices which can attain the Carnot COP.

Model (3) is non-ideal due to the delocalised dissipation effects.

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Model (3) is non-ideal due to the delocalised dissipation effects.

We can focus on the optimisation of a more sensible figure of merit: COP $\varepsilon_*$ at maximum cooling power.
Weak coupling to the baths: $\gamma \ll \{k_B T_\alpha, \hbar \omega_\alpha, g\}$

Born, Markov, and rotating wave approximations

Master equation: $\dot{\rho}(t) = (\mathcal{L}_W + \mathcal{L}_h + \mathcal{L}_c)\rho(t)$

Lindblad dissipators: $\mathcal{L}_\alpha = \sum_\omega (\propto \omega^d_\alpha) ...$

COP AT MAXIMUM POWER

\[ \varepsilon^* \leq \frac{d_c}{d_c + 1} \varepsilon_C \]

PERFORMANCE BOUND: $\varepsilon_* \leq \frac{d_c}{d_c+1} \varepsilon_C$

- Rigorously proven for models (1) and (2)
- Valid for any multistage refrigerator built upon (1)
- Verified numerically for model (3) as well
- Tight: saturated for $T_c \ll T_h, \omega_w \ll T_{w,h}$ (i.e. $\varepsilon_C \rightarrow 0$)

PERFORMANCE BOUND: \( \varepsilon_\star \leq \frac{d_c}{d_c+1} \varepsilon_C \)
The bound is clearly **model-independent** and holds for all known embodiments of quantum absorption fridges.

**IS IT UNIVERSAL?**
UNIVERSALITY: HEAT ENGINES

- Carnot efficiency: $\eta_C = 1 - \frac{T_c}{T_h}$
- **Endoreversible regime**: the main source of irreversibility is the imperfect thermal contact
- Effective temperature $T_h' \leq T_h$
- Efficiency at max power for endoreversible engines: $\eta_* = 1 - \sqrt{\frac{T_c}{T_h}}$
  - Yvon '55, Novikov '57; Curzon-Ahlborn '75
- When $\eta_C \to 0$: $\eta_* \approx \frac{1}{2} \eta_c + \frac{1}{8} \eta_c^2 + \cdots$
  - Van der Broeck, *Phys. Rev. Lett.* 95 (2005);
Endoreversible regime: the main source of irreversibility is the imperfect thermal contact ($T'_\alpha \neq T_\alpha$)

In the limit $T_c \ll T_h$, $\omega_w \ll T_{w,h}$ ...

COP at maximum power for all endoreversible refrigerators:

$$\varepsilon^* = \frac{\Lambda \varepsilon_C}{(1 - \Lambda)\varepsilon_C + 1}$$


But: $\Lambda$ depends on the bath details!

The COP bound cannot be universal

Model (1): Qutrit; $d_\alpha$-dimensional baths with flat spectral densities

We find: $\Lambda = d_c/(d_c + 1)$

Sharper performance bound (although strictly valid only at endoreversibility)

$$\varepsilon_* \leq \frac{d_c}{d_c + 1} + \varepsilon_C$$

TESTING THE BOUND: $\varepsilon_* \leq \frac{d_c}{d_c + 1 + \varepsilon_C} \varepsilon_C$

- $N$-stage quantum absorption refrigerators with three-dimensional unstructured baths ($d_\alpha = 3$)

ABSORPTION REFRIGERATORS

- How to understand and possibly improve their optimal performance?

CAN QUANTUMNESS HELP?
QUANTUM-ENHANCED FRIDGES

- Work bath: *squeezed thermal* (with squeezing degree $r$)
- Squeezing the 2$^{nd}$ law
  \[
  \frac{\dot{Q}_c}{T_c} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_w}{\tilde{T}_w(r)} \leq 0, \quad \tilde{T}_w(r) > T_w
  \]
- Modified master equation:
  \[
  \dot{\rho}(t) = \left( \mathcal{L}_w^{(r)} + \mathcal{L}_h + \mathcal{L}_c \right) \rho(t)
  \]
- The Carnot COP increases with $r$:
  \[
  \varepsilon_C(r) = \frac{1 - \frac{T_h}{\tilde{T}_w(r)}}{\frac{T_h}{T_c} - 1} > \varepsilon_C(0)
  \]

Squeezing the 2nd law

Squeezing the 2

QUANTUM-ENHANCED FRIDGES

\[ \varepsilon \]

\[ 10^2 \dot{Q}_c/\omega_h^2 \]

\[ T_h \]

\[ \gamma \]

\[ \omega_h \]

\[ \omega_w \]

\[ \omega_c \]

\[ \tilde{T}_w(r) \]

\[ T_c \]

Squeezing the 2nd law

Quantum engineering pushes refrigerator beyond classical efficiency limits
Overview of quantum refrigerators and their generic modelling using the framework of quantum tricycles.

Tight bound $\varepsilon^*/\varepsilon_C \leq d_c/(d_c + 1)$ on the coefficient of performance at maximum cooling power for all known models of quantum absorption refrigerators.


Quantum fluctuations in the work bath (e.g. squeezing) can push the performance beyond classical limitations.
WHAT IS GENUINELY QUANTUM IN QUANTUM THERMODYNAMICS?

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L. A. Correa, J. P. Palao, GA & D. Alonso
Performance bound for quantum absorption refrigerators

L. A. Correa, J. P. Palao, D. Alonso & GA
Quantum-enhanced absorption refrigerators

L. A. Correa, J. P. Palao, GA & D. Alonso
Optimal performance of endoreversible quantum refrigerators

THANK YOU

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