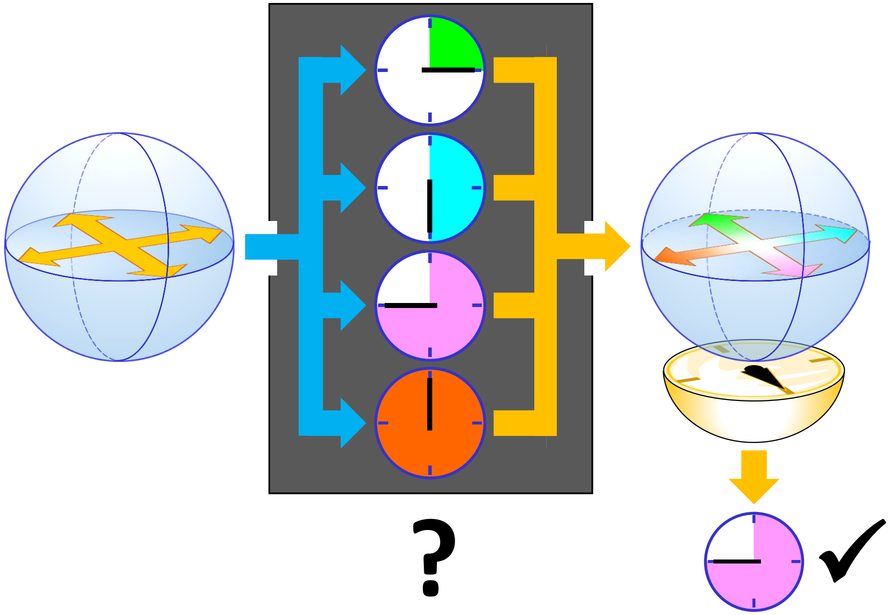
Robustness of Coherence

an operational and observable measure of quantum coherence



Quantum Correlations Group
The University of Nottingham

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Robustness of Coherence

an operational and observable measure of quantum coherence

Gerardo Adesso

joint work with

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N. Johnston *(Mount Allison, Canada)*, M. Piani *(Strathclyde, UK)*

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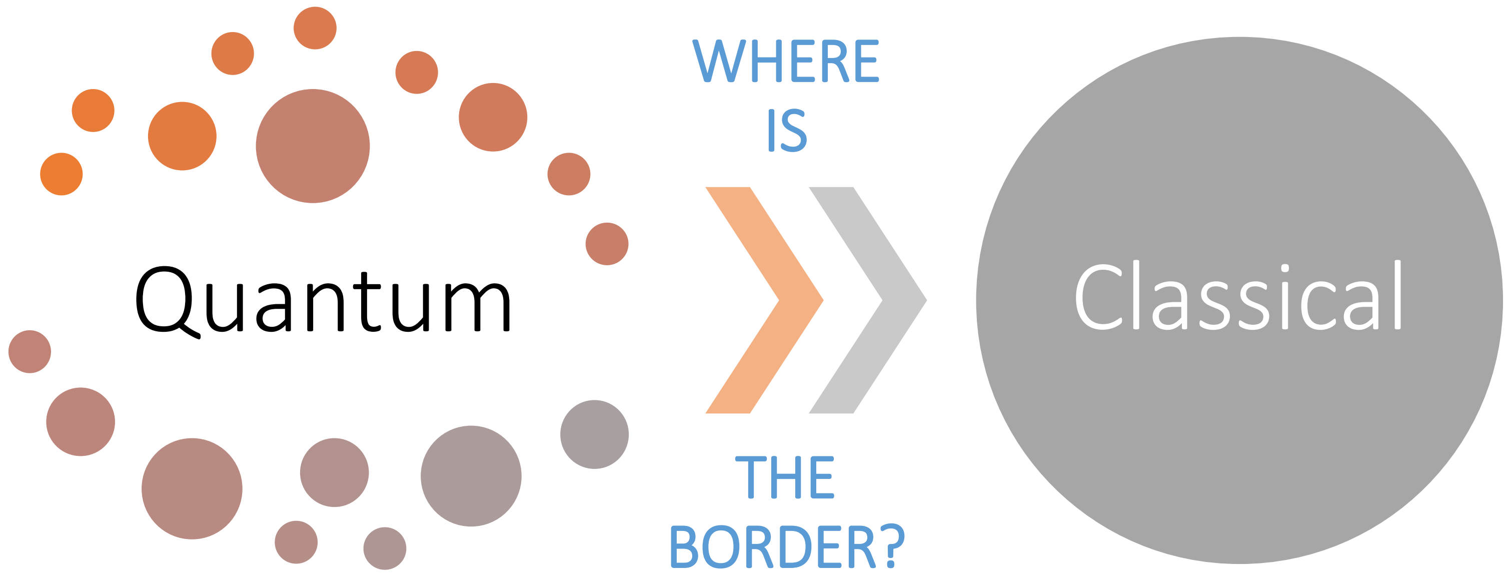
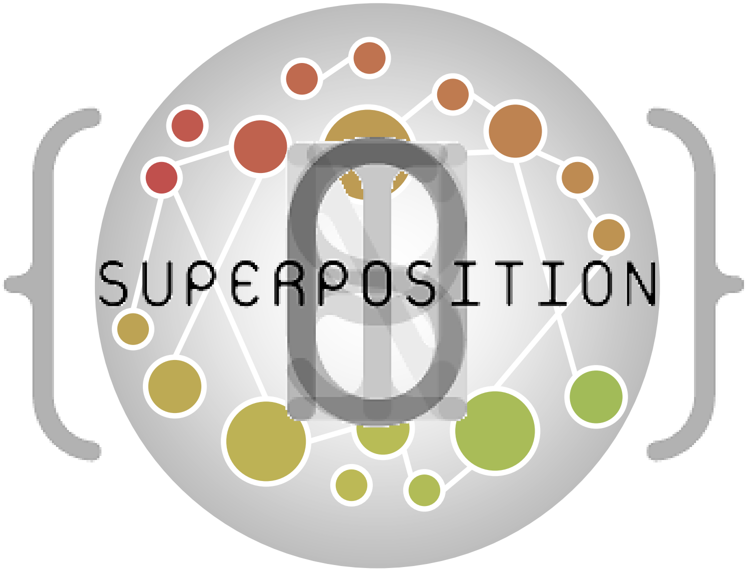
MARIE CURIE ACTIONS

Our research at Nottingham



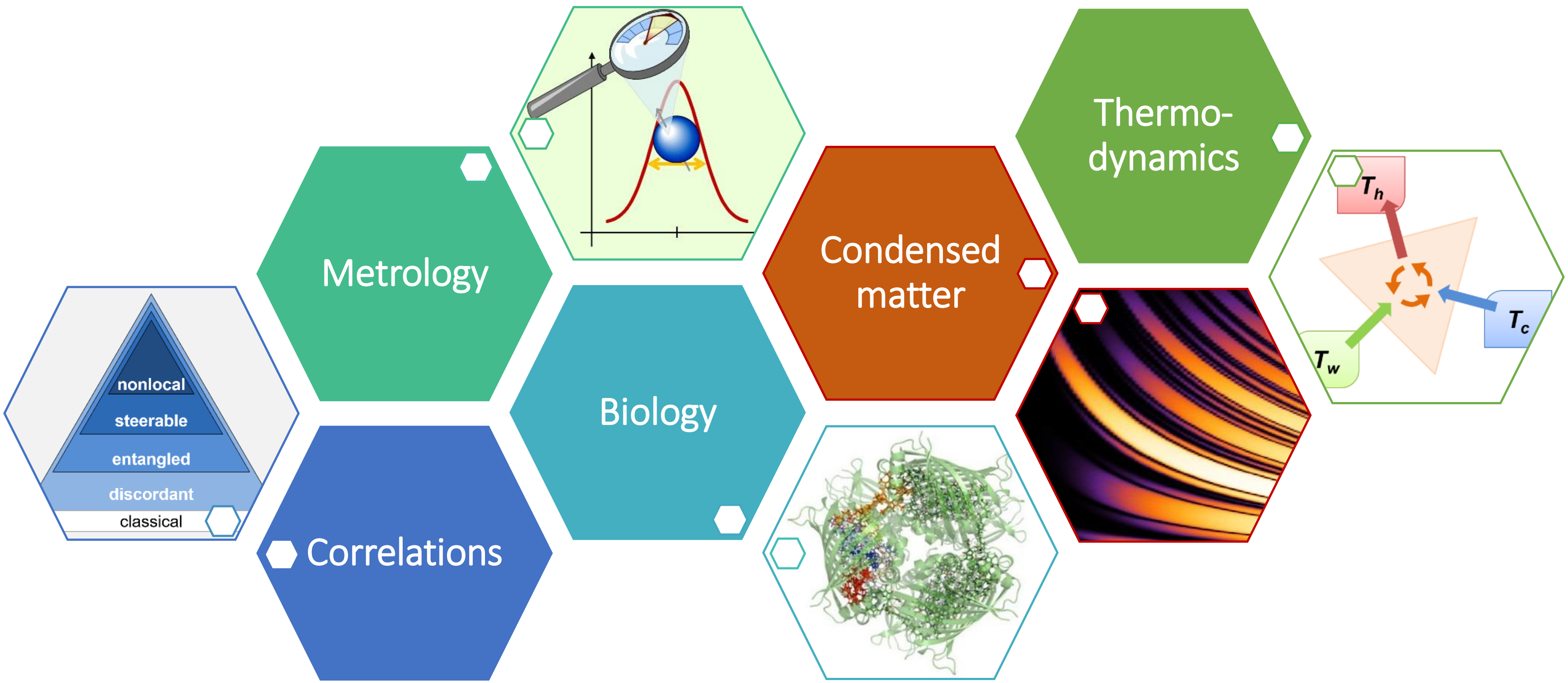
- Identifying quantumness by its most essential and genuine signatures in general composite systems
- Providing novel operational interpretations and satisfactory measures for quantum resources

Quantum coherence



- The possibility to create "quantum" superpositions of a reference set of "classical" orthogonal states
- Quintessential feature of quantum mechanics and key ingredient for quantum technologies

Quantum coherence

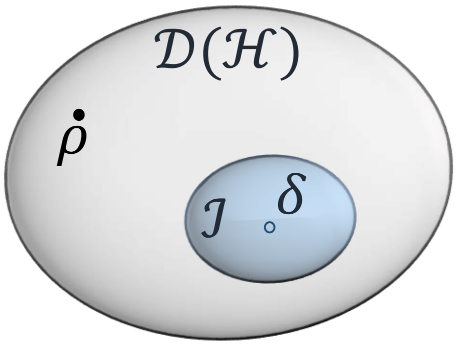


A fully satisfactory (mathematically rigorous and physically intuitive) quantitative **measure of coherence** with a direct **operational interpretation** is still rare to find!

This talk

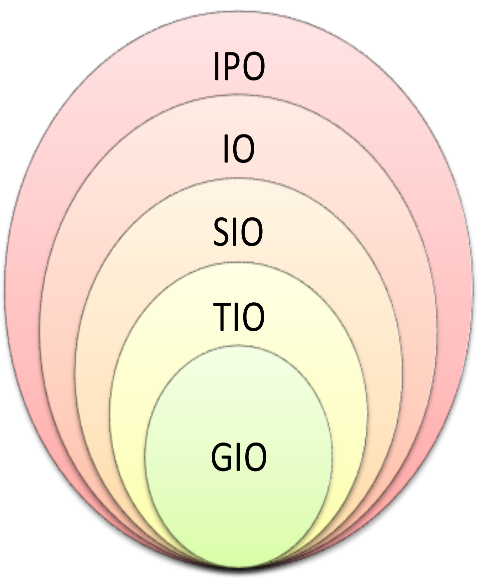
- *Characterising coherence*
  Quantum Resource Theories
- *Quantifying coherence*
  Robustness of Coherence
- *Measuring coherence*
  Coherence Witnesses
- *Exploiting coherence*
  Phase Discrimination Games

Resource theories of coherence



Free states

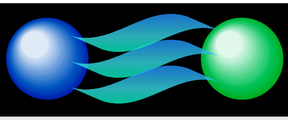
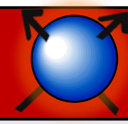
- **Incoherent states**: States diagonal in the reference basis $\{|j\rangle\}$: $\delta \in \mathcal{I}$: $\delta = \sum_j p_j |j\rangle\langle j|$



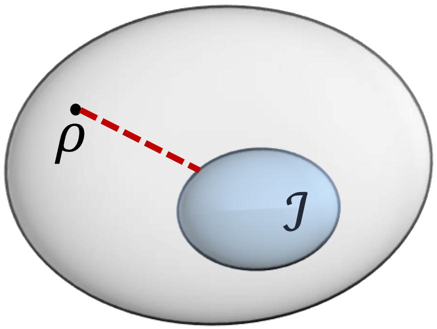
Free operations

- Operations unable to create coherence, that map incoherent states into incoherent states
- **Incoherence-preserving operations** *(Aaberg 2006)* $\supset$ **incoherent operations** *(Baumgratz et al 2014)* $\supset$ **strictly incoherent operations** *(Yadin et al 2015)* $\supset$ **translationally invariant operations** *(Marvian et al 2015)* $\supset$ **genuinely incoherent operations** *(Streltsov 2015)*

Resource theories of coherence

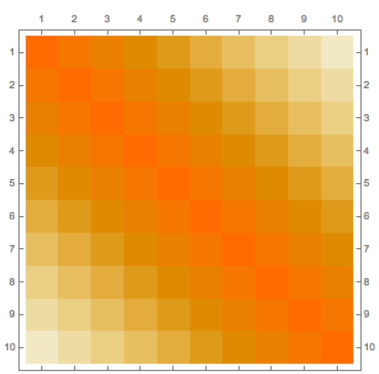
Entanglement E 	**Coherence C** 
with respect to a bipartition $A\!:\!B$	*with respect to a fixed reference basis $\{\|j\rangle\}$*
(E1) $E_{A:B}(\sigma_{AB}) = 0$ for separable states $\sigma_{AB} \in \mathcal{S}$: $\sigma_{AB} = \sum_k p_k \rho_A^k \otimes \nu_B^k$	(C1) $C(\delta) = 0$ for incoherent states $\delta \in \mathcal{I}$: $\delta = \sum_j p_j \|j\rangle\langle j\|$
(E2) $E_{A:B}$ is nonincreasing under LOCC on A and B, i.e. acting as $\Lambda_{LOCC}[\rho] = \sum_j L_j \rho L_j^\dagger$, with L_j being the Kraus operators (properties omitted here) *[R., P., M., and K. Horodecki, Rev. Mod. Phys. 81, 865 (2009)]*	(C2) C is nonincreasing under free operations, e.g. incoherent operations *[Baumgratz et al PRL 2014]* acting as $\Lambda_{\mathcal{I}}[\rho] = \sum_j K_j \rho K_j^\dagger$, with incoherence-preserving Kraus operators $K_j \mathcal{I} K_j^\dagger \subseteq \mathcal{I}$
(E2b) $E_{A:B}$ is nonincreasing on average under selective LOCC, i.e. $E_{A:B}(\rho) \geq \sum_j p_j E_{A:B}(\rho_j)$ with $\rho_j = L_j \rho L_j^\dagger / p_j$	(C2b) C is nonincreasing on average under selective incoherent operations, i.e. $C(\rho) \geq \sum_j p_j C(\rho_j)$ with $\rho_j = K_j \rho K_j^\dagger / p_j$
(E3) $E_{A:B}$ is convex (optional)	(C3) C is convex (optional)

Quantifying coherence



Relative entropy

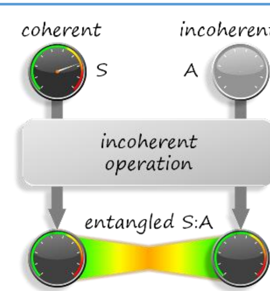
- Distance from the set of incoherent states
- $C_{RE}(\rho) = \inf_{\delta \in \mathcal{J}} S(\rho || \delta) = S\big(\Delta(\rho)\big) - S(\rho)$, with $\Delta(\rho)$ the diagonal part of ρ



l1-norm

- Contribution of the off-diagonal entries of the density matrix
- $C_{l1}(\rho) = \sum_{k \neq j} |\rho_{jk}|$

- Several other proposals to quantify coherence (e.g. in terms of **entanglement creation**), …

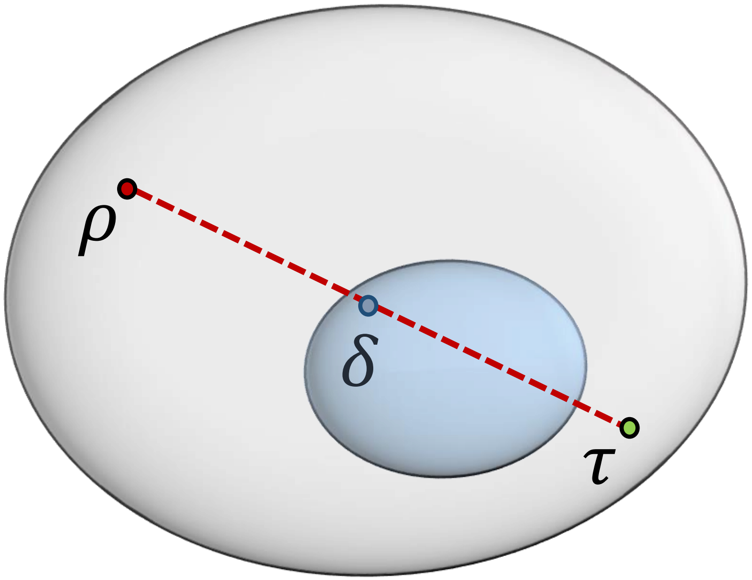


(A. Streltsov et al, ***Rev. Mod. Phys.*** *in progress)*

Robustness of Coherence

Definition (RoC)

- $C_{\mathcal{R}}(\rho) = \min_{\tau} \left\{ s \geq 0 \mid \frac{\rho + s\,\tau}{1+s} =: \delta \in \mathcal{I} \right\}$

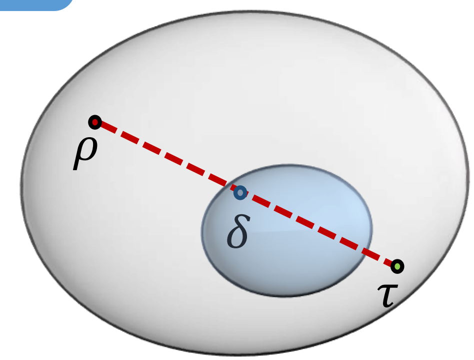


The RoC of a quantum state ρ quantifies the minimum weight s of another state τ such that its convex mixture with ρ yields an incoherent state δ

(notion previously used for entanglement)

Robustness of Coherence

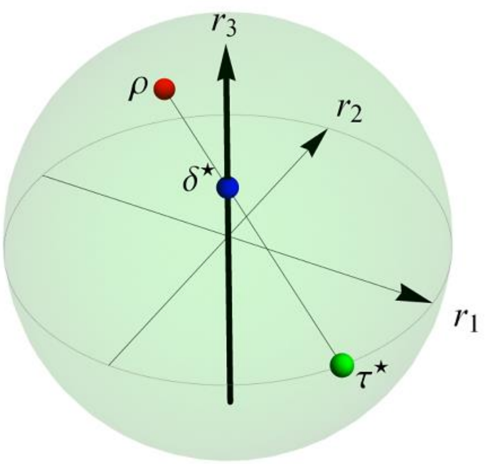
Properties (RoC)



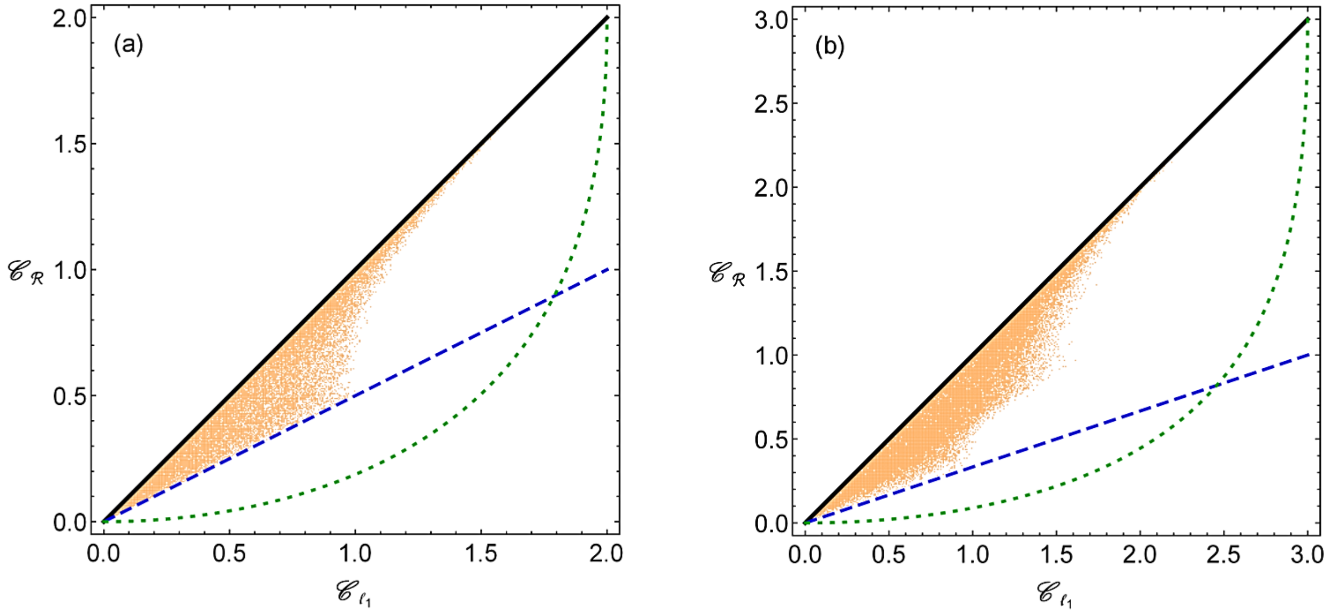
- Vanishing iff a state is incoherent
- Full monotone in *all* possible resource theories of coherence
  - $C_{\mathcal{R}}(\rho) \geq \sum_l \mathrm{Tr}[\Gamma_l(\rho)]\, C_{\mathcal{R}}\left(\frac{\Gamma_l(\rho)}{\mathrm{Tr}[\Gamma_l(\rho)]}\right)$
- Convex
- Computable numerically by a semidenite program (SDP) for arbitrary states [Matlab code provided]
- Computable analytically for all one-qubit states, all pure *d*-dimensional states, and a class of mixed states

Robustness of Coherence: Examples

RoC for one qubit
(= l1-norm)



RoC vs l1-norm of coherence in dimensions (a) ***d*=3** and (b) ***d*=4**



$$\frac{C_{l1}(\rho)}{d-1} \leq C_{\mathcal{R}}(\rho) \leq C_{l1}(\rho)$$

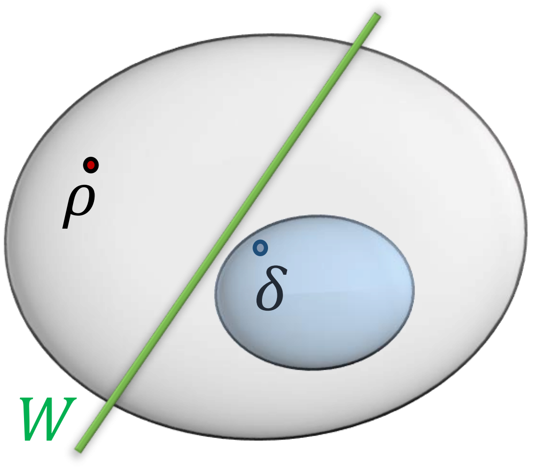
Coherence Witnesses

Definition (**Coherence Witnesses**)

- Any Hermitian operator W with nonnegative diagonal $\Delta(W) \geq 0$ in the reference basis is a **coherence witness** :
  - $\mathrm{Tr}(\delta W) \geq 0$ for all incoherent states δ
  - If one measures the observable W on the state ρ and finds $\mathrm{Tr}(\rho W) < 0$, then coherence of ρ is detected

Witnesses can be very useful to detect coherence in experiments

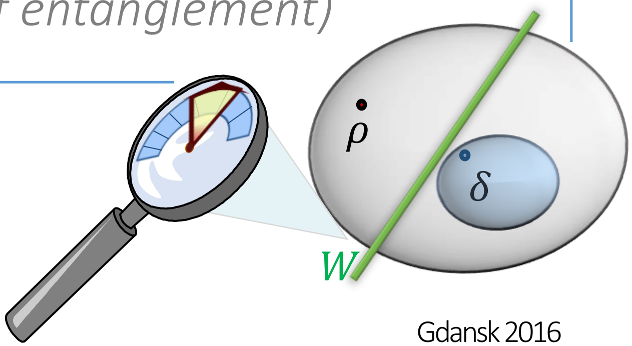
(notion previously used for entanglement)



Coherence Witnesses and RoC

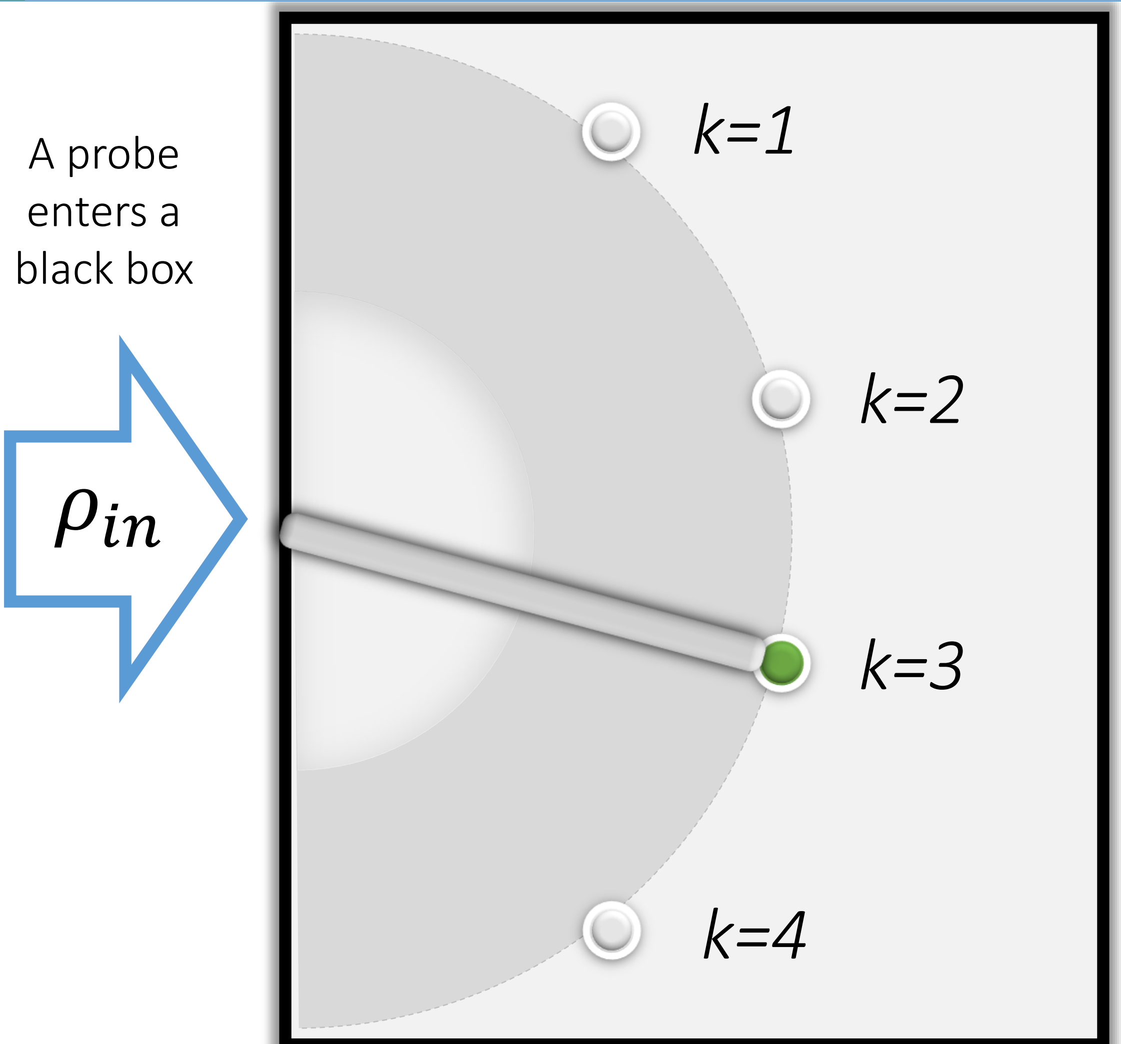
The RoC is an **observable** measure

- Coherence witnesses give **lower bounds** to the RoC:
  $C_{\mathcal{R}}(\rho) \geq \max\{0, -\mathrm{Tr}(\rho W)\} \quad \forall\, W \leq \boldsymbol{I}, \Delta(W) \geq 0$
- The bounds can be saturated: the RoC is **observable** (note the optimal witness W is dependent on ρ)
- Alternatively, one can provide **state-independent** accessible lower bounds, and estimation procedures for the RoC based on incomplete **experimental data**
  (all formulated as simple SDPs, unlike the case of entanglement)

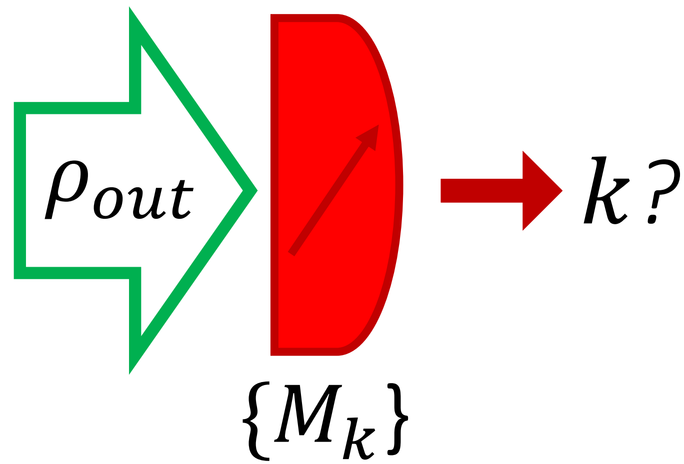


Phase Discrimination Games

A probe
enters a
black box



The black box randomly applies a phase shift ϕ_k sampled from an agreed set of choices $\{\phi_k\}$ with prior probability distribution $\{p_k\}$



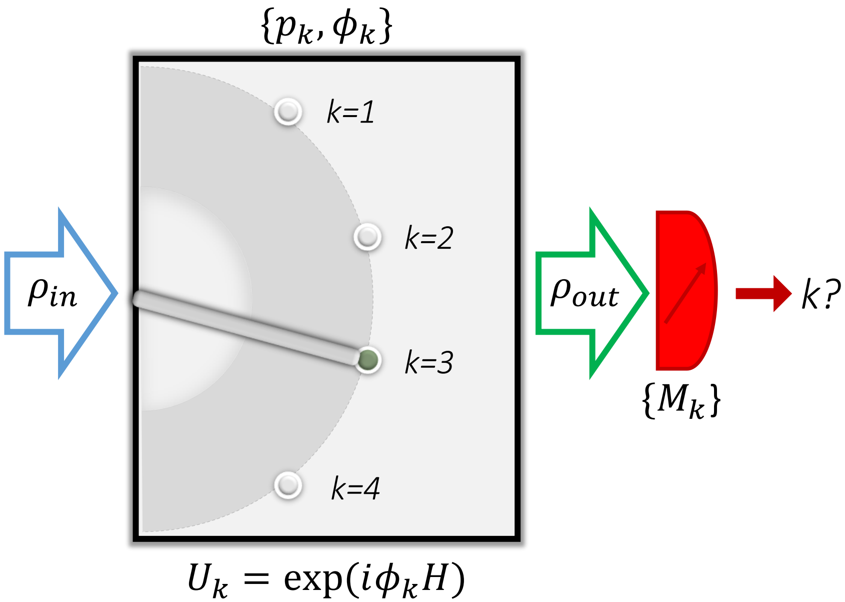
An optimal POVM with elements $\{M_k\}$ is performed on the output to guess the correct instance of the phase applied to the probe…

$$U_k = \exp(i\phi_k H) \text{ with } H = \sum_j j\,|j\rangle\langle j|$$

Phase Discrimination Games

Probability of success

- Each setting $\Theta \coloneqq \{p_k, \phi_k\}$ defines an instance of phase discrimination (PD) game
- The maximal probability of guessing right (probability of success) for a given input is: $p_\Theta^{succ}(\rho_{in}) = \max_{\{M_k\}} \sum_k p_k \mathrm{Tr}[U_k \rho_{in} U_k^\dagger M_k]$
- Incoherent probes $\delta_{in} \in \mathcal{J}$ are of no help! $p_\Theta^{succ}(\mathcal{J}) = \max_k p_k$ (according to just casting a guess on the most likely phase)
- ***Quantitatively, how better can probes with coherence be for this task?***

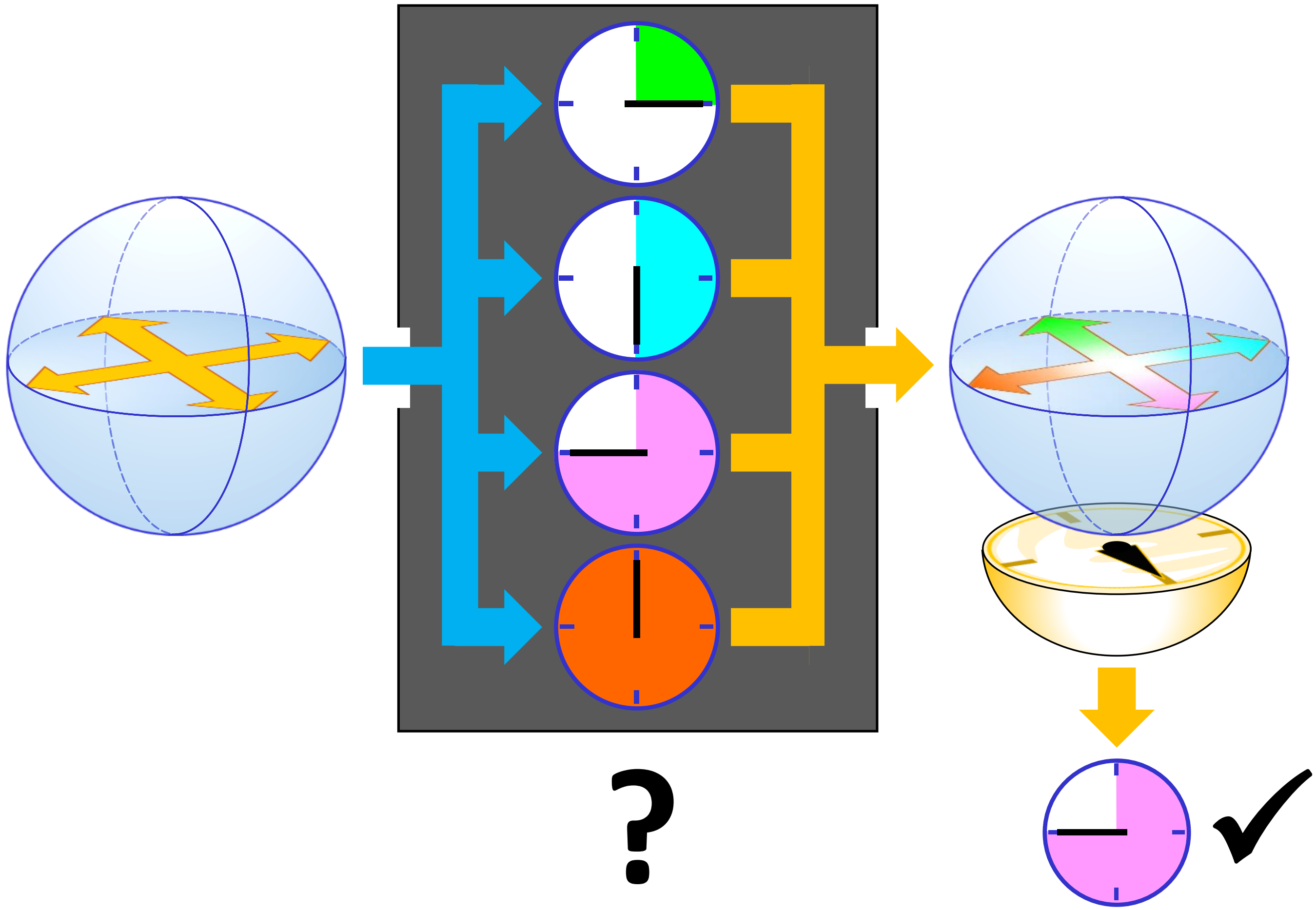


$$\max_\Theta \frac{p_\Theta^{succ}(\rho_{in})}{p_\Theta^{succ}(\mathcal{J})} = 1 + C_\mathcal{R}(\rho_{in})$$

The RoC quantifies the advantage in PD games

Physicists quantify the usefulness of 'quantum weirdness'

April 13, 2016 by Lisa Zyga, Phys.org report



A scheme of a phase discrimination task. The "robustness of coherence" of the input probe quantifies the quantum advantage. Credit: Gerardo Adesso, University of Nottingham

Summary

The quest for fully characterising quantum ***coherence as a resource*** *is still in progress*

We have introduced the ***Robustness of Coherence****, a full convex monotone in all possible resource theories of coherence*

We have introduced ***coherence witnesses****, useful for experiments, and shown that the Robustness is an* ***observable*** *quantity*

We have provided a direct ***operational interpretation*** *for the Robustness as the advantage in* ***phase discrimination games***

Thank you

The University of Nottingham

C. Napoli *et al.* Phys. Rev. Lett. **116**, 150502 (2016) Editors' Suggestion
M. Piani *et al.* Phys. Rev. A **93**, 042107 (2016) Editors' Suggestion

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