OPTIMAL QUANTUM TELEPORTATION WITH LIMITED RESOURCES

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SCUOLA NORMALE SUPERIORE

QUANTUM TELEPORTATION

is a fundamental building block for quantum communication

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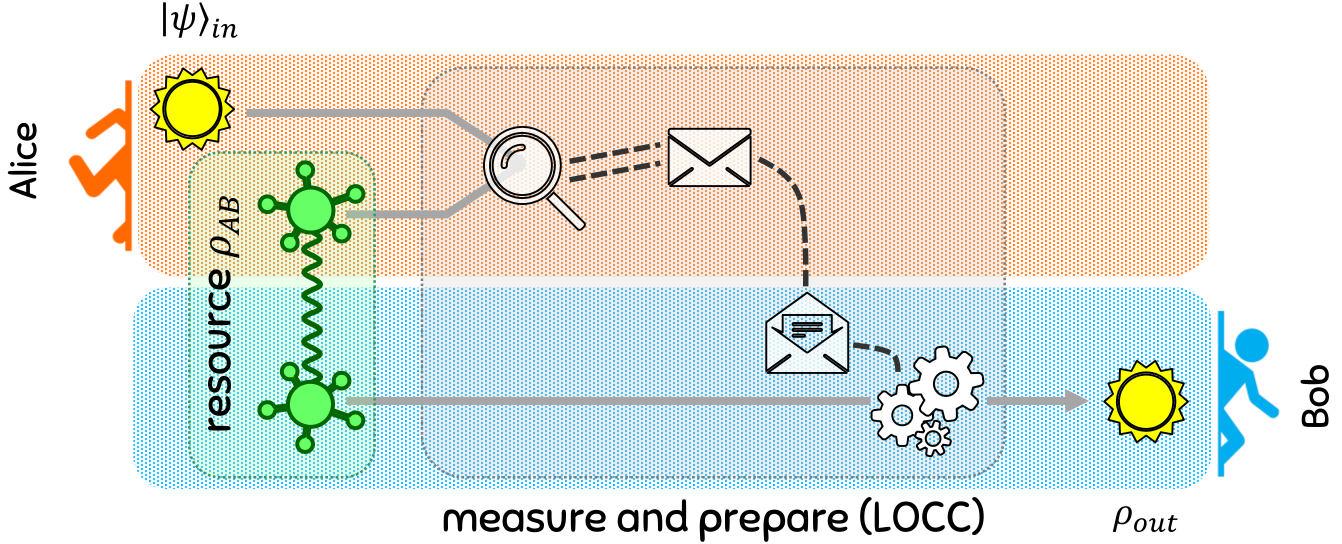
(mathematically) is the implementation of an identity channel

QUANTUM TELEPORTATION

If ρ_{AB} is maximally entangled

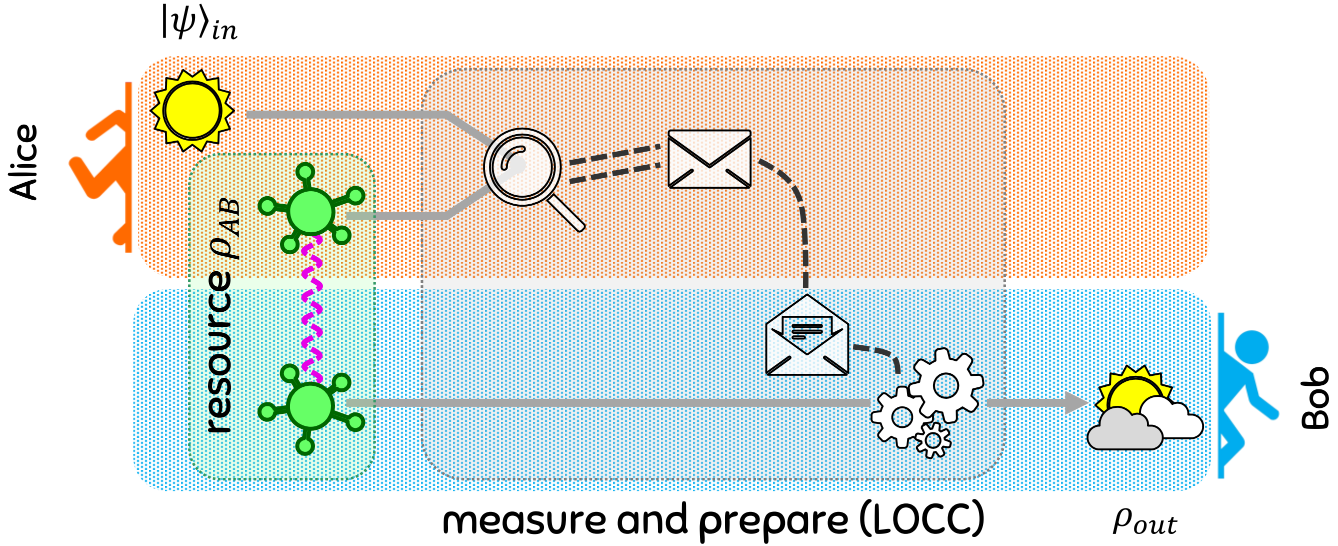
Fidelity $\mathcal{F} = {}_{in}\langle\psi|\rho_{out}|\psi\rangle_{in} = 1$



QUANTUM TELEPORTATION

If ρ_{AB} is not maximally entangled

Fidelity $\mathcal{F} = {}_{in}\langle\psi|\rho_{out}|\psi\rangle_{in} < 1$



WHAT IS THE BEST WE CAN DO WITH A FINITE AMOUNT OF ENTANGLEMENT?

FOR DISCRETE VARIABLES: FIDELITY PROPORTIONAL TO MAX SINGLET FRACTION

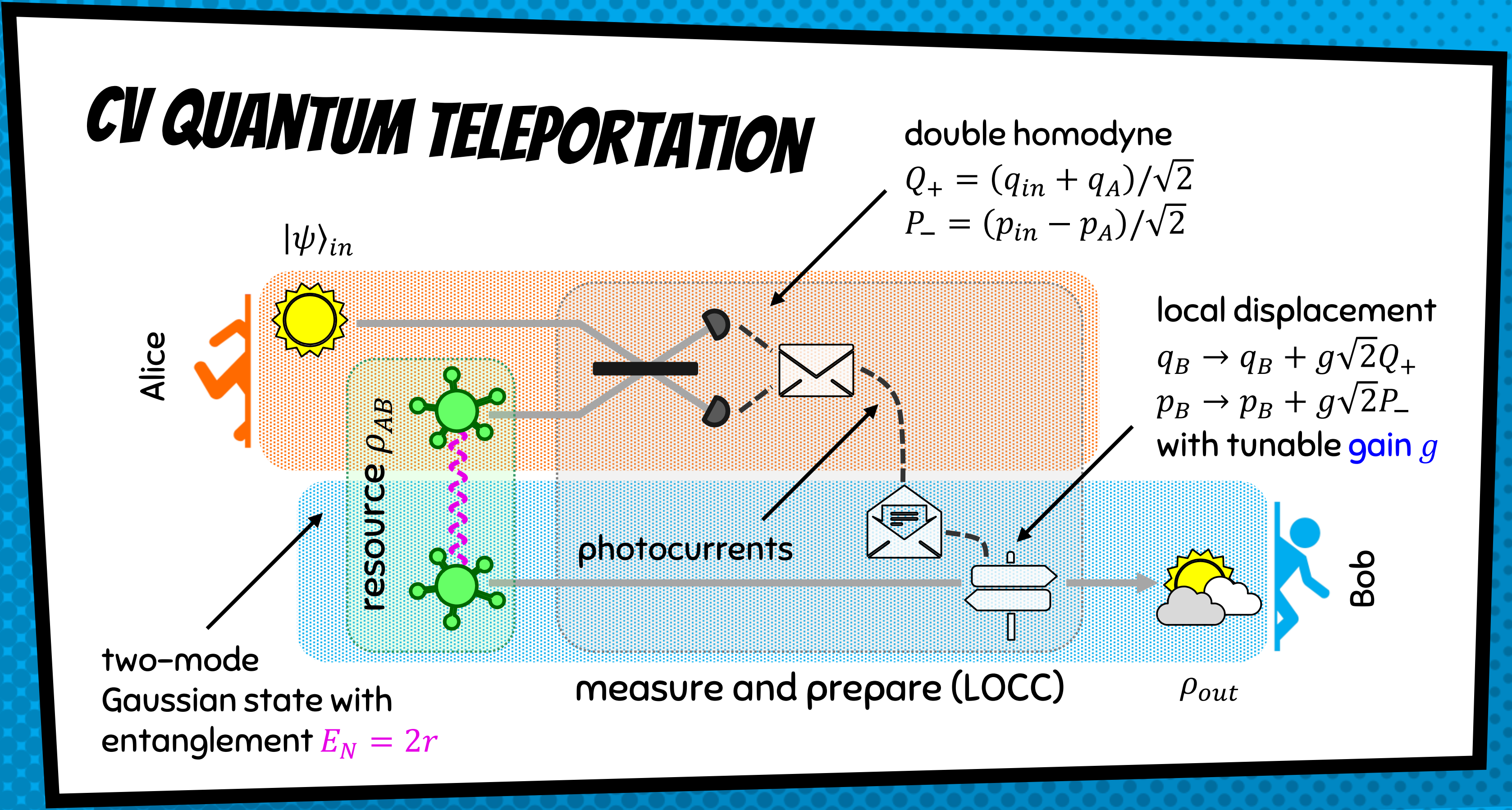
(Horodecki x3 1999)

WHAT ABOUT CONTINUOUS VARIABLE SYSTEMS?

1.

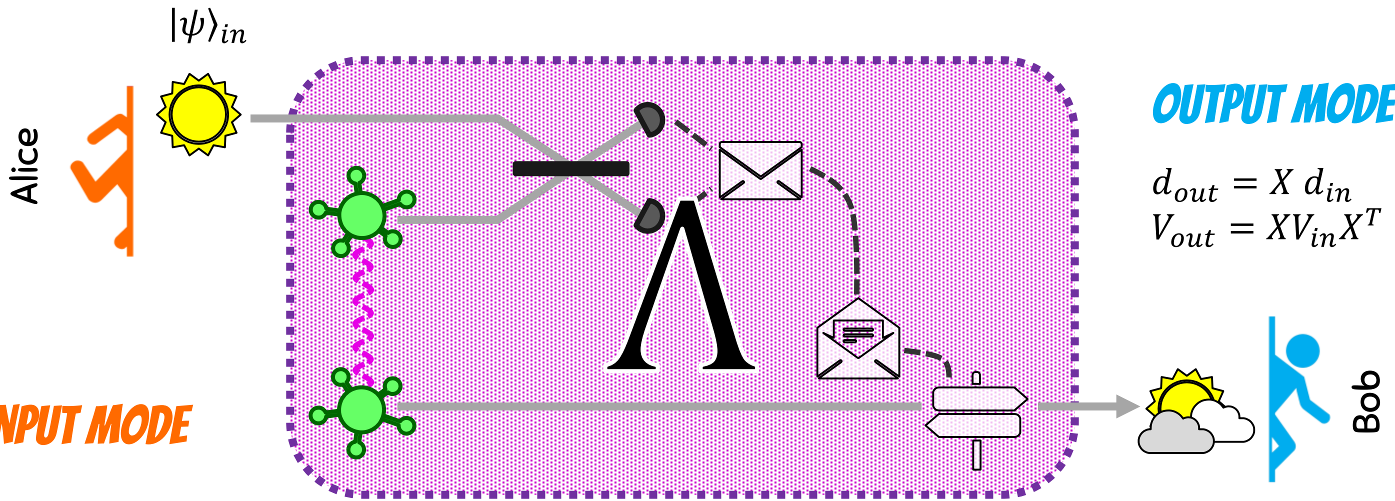
CONTINUOUS VARIABLE (CV) TELEPORTATION

Braunstein–Kimble protocol



CV QUANTUM TELEPORTATION
double homodyne
$Q_+ = (q_{in} + q_A)/\sqrt{2}$
$P_- = (p_{in} - p_A)/\sqrt{2}$
$|\psi\rangle_{in}$
Alice
resource ρ_{AB}
local displacement
$q_B \to q_B + g\sqrt{2}Q_+$
$p_B \to p_B + g\sqrt{2}P_-$
with tunable gain g
photocurrents
Bob
two-mode
Gaussian state with
entanglement $E_N = 2r$
measure and prepare (LOCC)
ρ_{out}

CV QUANTUM TELEPORTATION AS A GAUSSIAN CHANNEL



Gaussian channel $\Lambda \equiv (X, Y)$

INPUT MODE

d_{in} (1st moments)
V_{in} (covariance matrix)

OUTPUT MODE

$$d_{out} = X\, d_{in}$$
$$V_{out} = X V_{in} X^T + Y$$

CV QUANTUM TELEPORTATION AS A GAUSSIAN CHANNEL

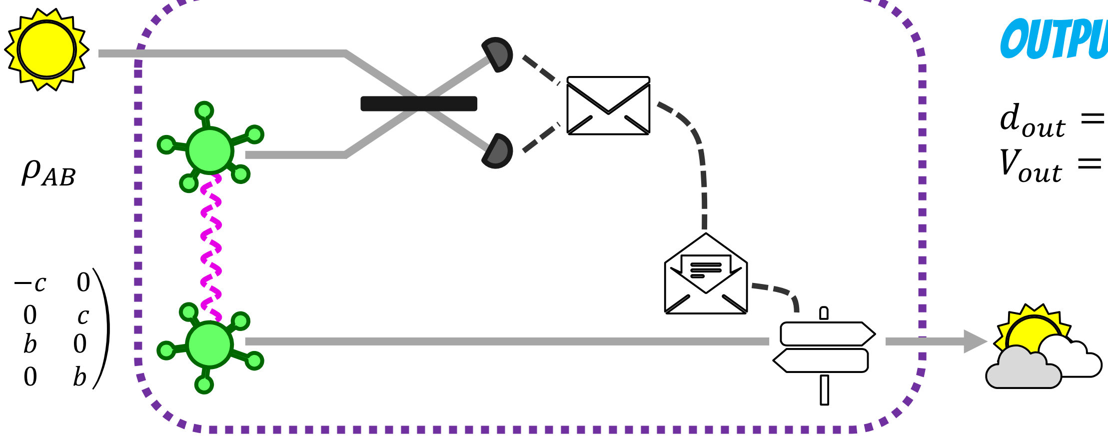
INPUT $|\psi\rangle_{in}$

d_{in}
V_{in}

RESOURCE ρ_{AB}

$$d_{AB} = 0$$

$$V_{AB} = \begin{pmatrix} a & 0 & -c & 0 \\ 0 & a & 0 & c \\ -c & 0 & b & 0 \\ 0 & c & 0 & b \end{pmatrix}$$



OUTPUT ρ_{out}

$$d_{out} = X\, d_{in}$$
$$V_{out} = X V_{in} X^T + Y$$

Phase-insensitive Gaussian channel $\Lambda \equiv (X, Y)$ with:

$$X = g\mathbb{I}, \quad Y = (g^2 a - 2gc + b)\mathbb{I}$$

2.

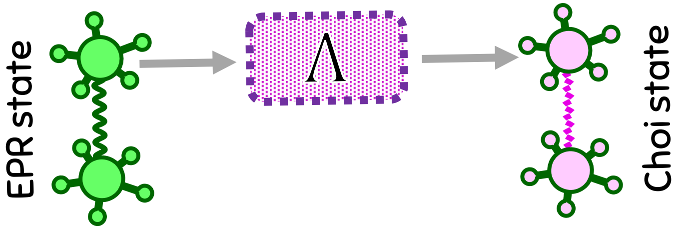
Phase-insensitive Gaussian channels

implementable by teleportation

IMPLEMENTABLE PHASE-INSENSITIVE CHANNELS

- Entanglement of Gaussian states cannot be distilled by Gaussian LOCC (Giedke-Cirac 2002)
- All phase-insensitive Gaussian channels are teleportation-covariant (Pirandola VS Wilde 2015-17)

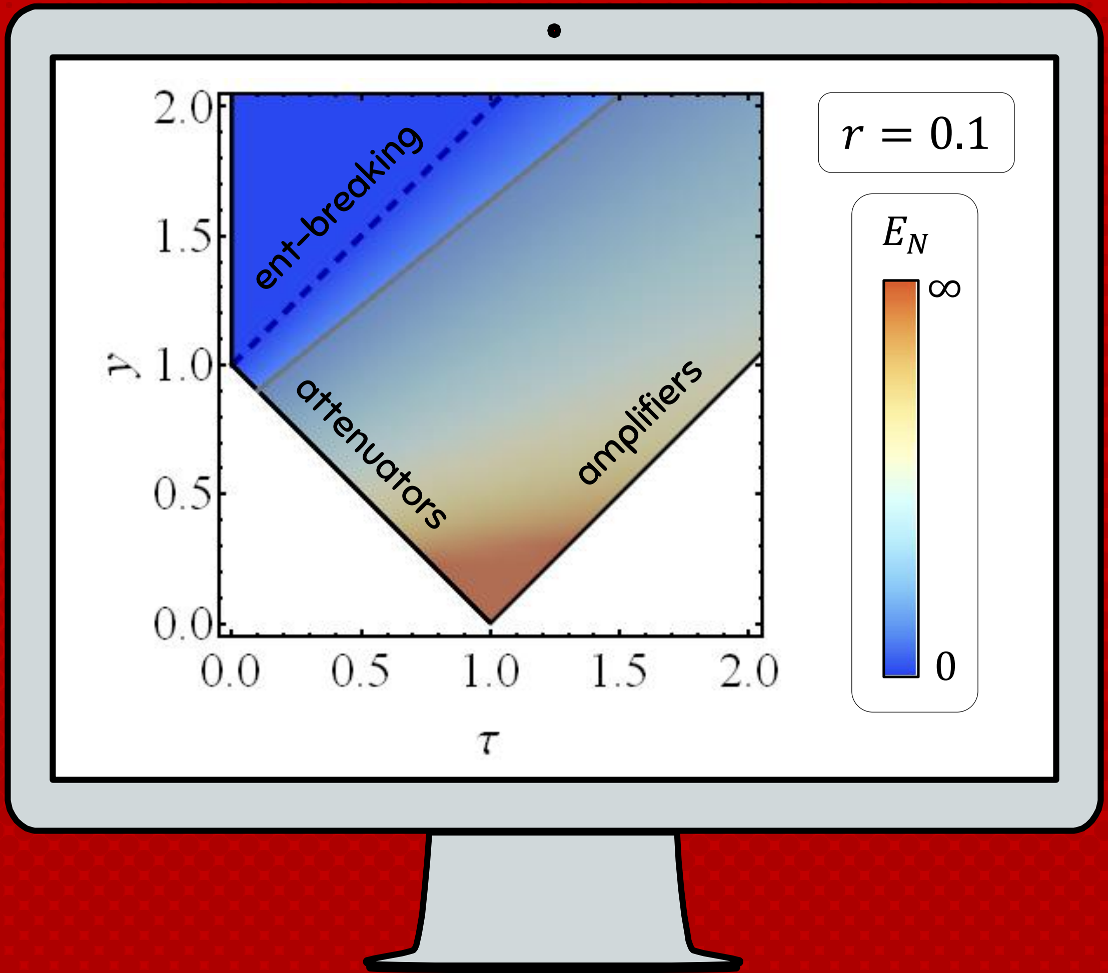
Implementable channels iff: $E_N(V_{AB}) \geq E_N(V_{Choi})$



PHASE-INSENSITIVE GAUSSIAN CHANNELS

$$X = \sqrt{\tau}\,\mathbb{I}, \quad Y = y\mathbb{I}$$

$y \geq \lvert 1 - \tau \rvert$	Completely positive
$y \geq 1 + \tau$	Entanglement-breaking
$y \geq e^{-2r}(1 + \tau)$	Implementable with $E_N = 2r$



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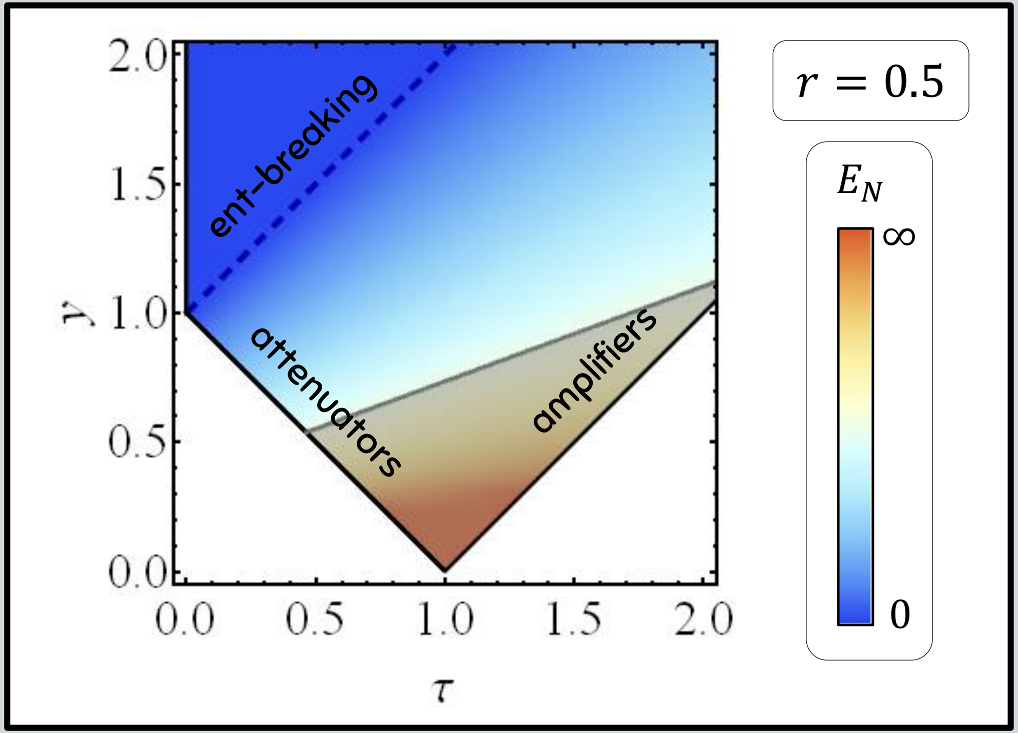

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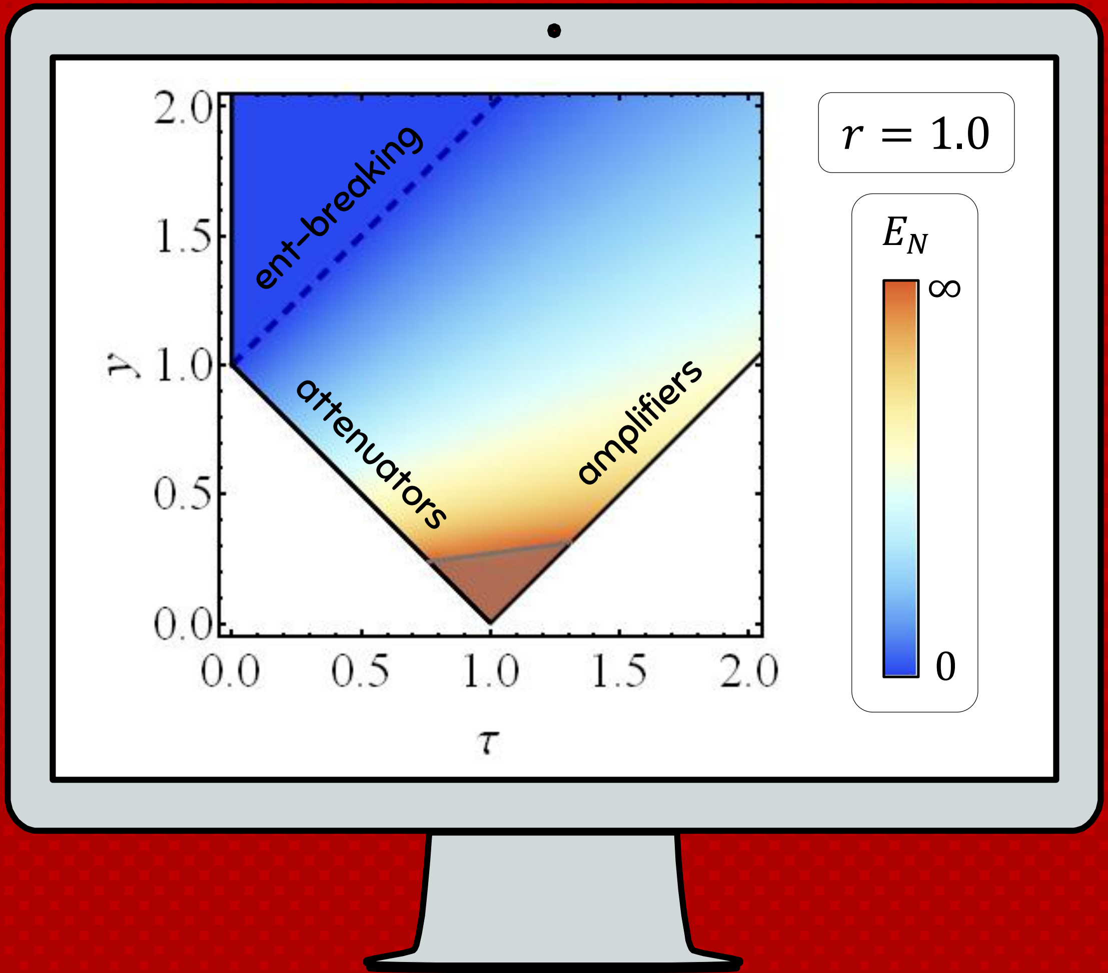
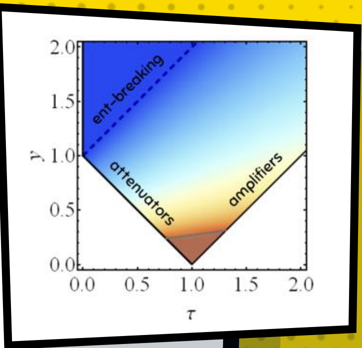
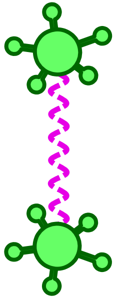


Implementable phase-insensitive channels



- Optimal resources given $X = \sqrt{\tau}\,\mathbb{I}, \quad Y = y\mathbb{I}$



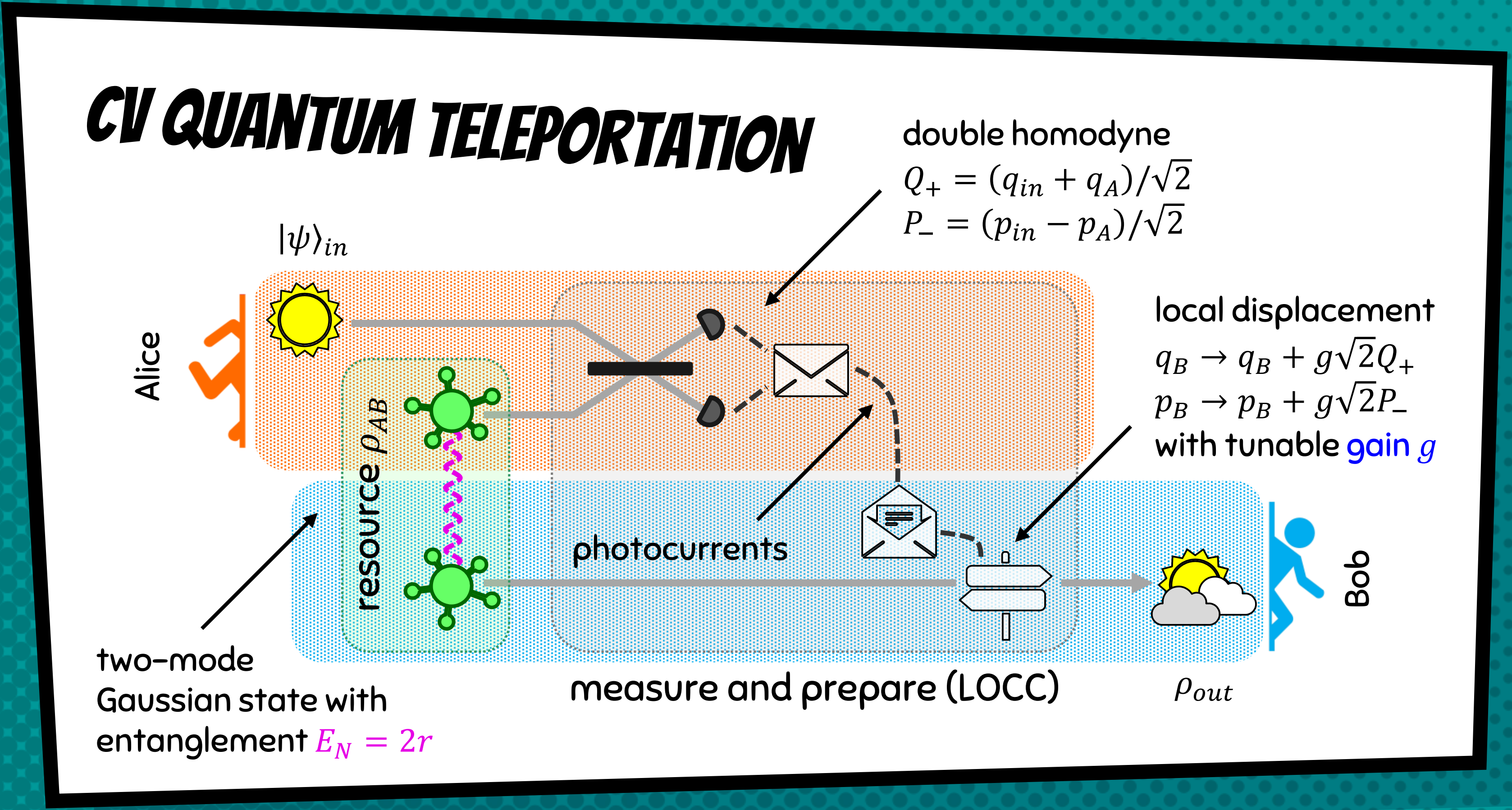
$$\begin{pmatrix} a & 0 & -c & 0 \\ 0 & a & 0 & c \\ -c & 0 & b & 0 \\ 0 & c & 0 & b \end{pmatrix} \quad \text{with} \quad b = \frac{\tau - e^{-2r}\tanh r}{\tau - \tanh r}, \quad a = \frac{b + e^{-2r}(\tau - 1)}{\tau}, \quad c = \frac{b - e^{-2r}}{\sqrt{\tau}}$$

- Minimum entanglement & finite mean energy (except for the quantum-limited attenuators)

3.

OPTIMAL AVERAGE TELEPORTATION FIDELITY

for an alphabet of coherent states



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CV QUANTUM TELEPORTATION OF COHERENT STATES

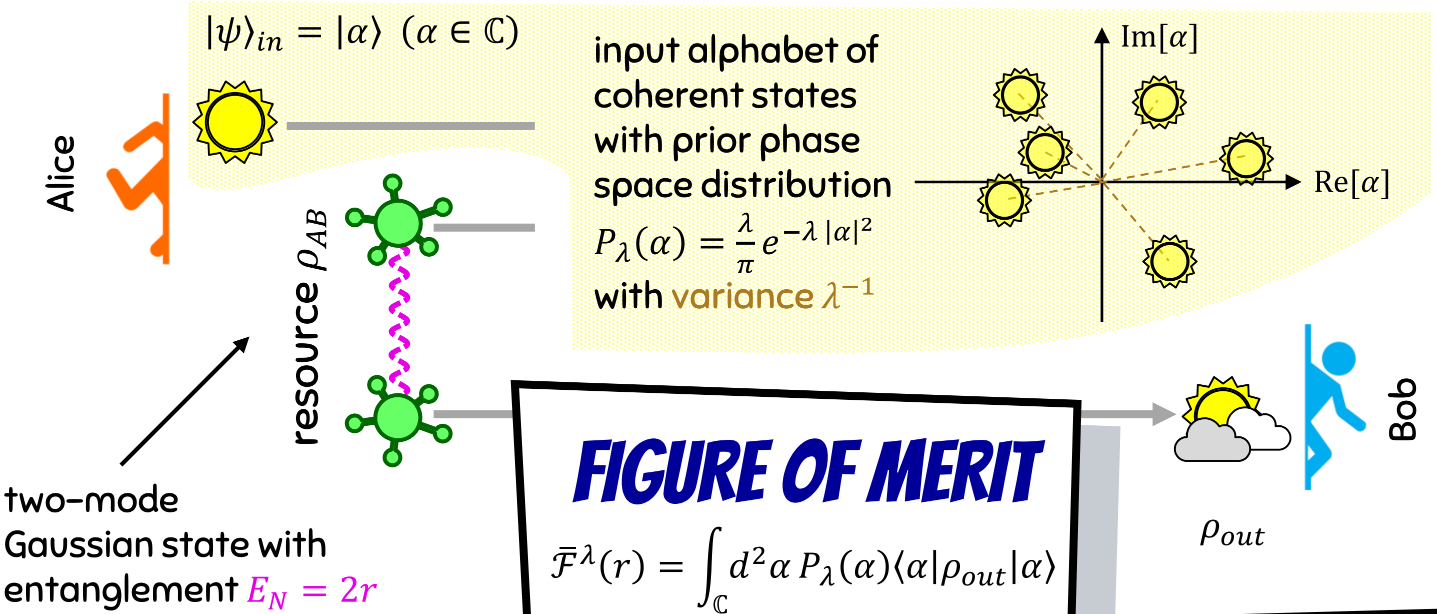


FIGURE OF MERIT

$$\bar{\mathcal{F}}^{\lambda}(r) = \int_{\mathbb{C}} d^2\alpha \, P_\lambda(\alpha) \langle\alpha|\rho_{out}|\alpha\rangle$$

average teleportation fidelity

FIGURE OF MERIT

$$\bar{\mathcal{F}}^{\lambda}(r) = \int_{\mathbb{C}} d^2\alpha \, P_{\lambda}(\alpha) \langle \alpha | \rho_{out} | \alpha \rangle$$

OPTIMAL FIDELITY: KNOWN CASES

$r = 0$ (separable resource)

$$\bar{\mathcal{F}}^{\lambda}_{\text{opt}}(0) = \frac{1+\lambda}{2+\lambda}$$

(Braunstein et al 2000, Hammerer et al 2005)

– Classical benchmark

– Optimal protocol: Heterodyne measure & prepare with gain $g = (1+\lambda)^{-1}$

$\lambda \to 0$ (flat input distribution)

$$\bar{\mathcal{F}}^{0}_{\text{opt}}(r) = \frac{1}{1+e^{-2r}}$$

(Adesso–Illuminati 2005, Mari–Vitali 2008)

– Optimal resource state: Pure two-mode squeezed state

– Optimal protocol: Standard BK with unit gain

FIGURE OF MERIT

$$\bar{\mathcal{F}}^{\lambda}(r) = \int_{\mathbb{C}} d^2\alpha \, P_{\lambda}(\alpha) \langle\alpha|\rho_{out}|\alpha\rangle$$

OPTIMAL FIDELITY: KNOWN CASES

$\bar{\mathcal{F}}^{\lambda\to 0}$

needs entanglement in ρ_{AB} needs steering in ρ_{AB}

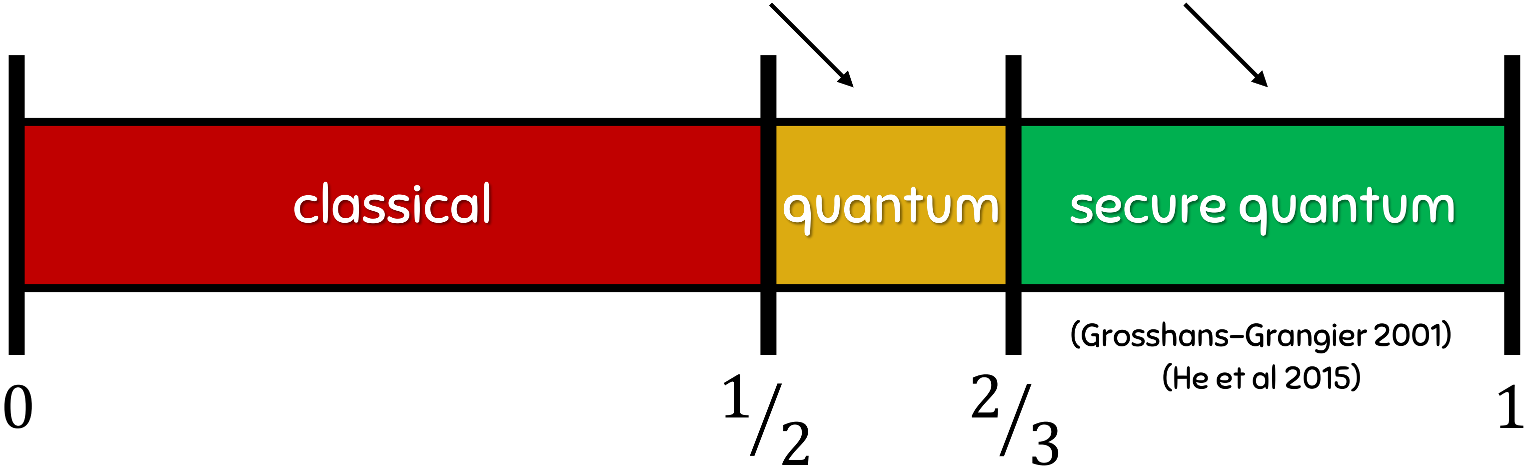


Figure of merit

$$\bar{\mathcal{F}}^{\lambda}(r) = \int_{\mathbb{C}} d^2\alpha\, P_{\lambda}(\alpha)\langle\alpha|\rho_{out}|\alpha\rangle$$

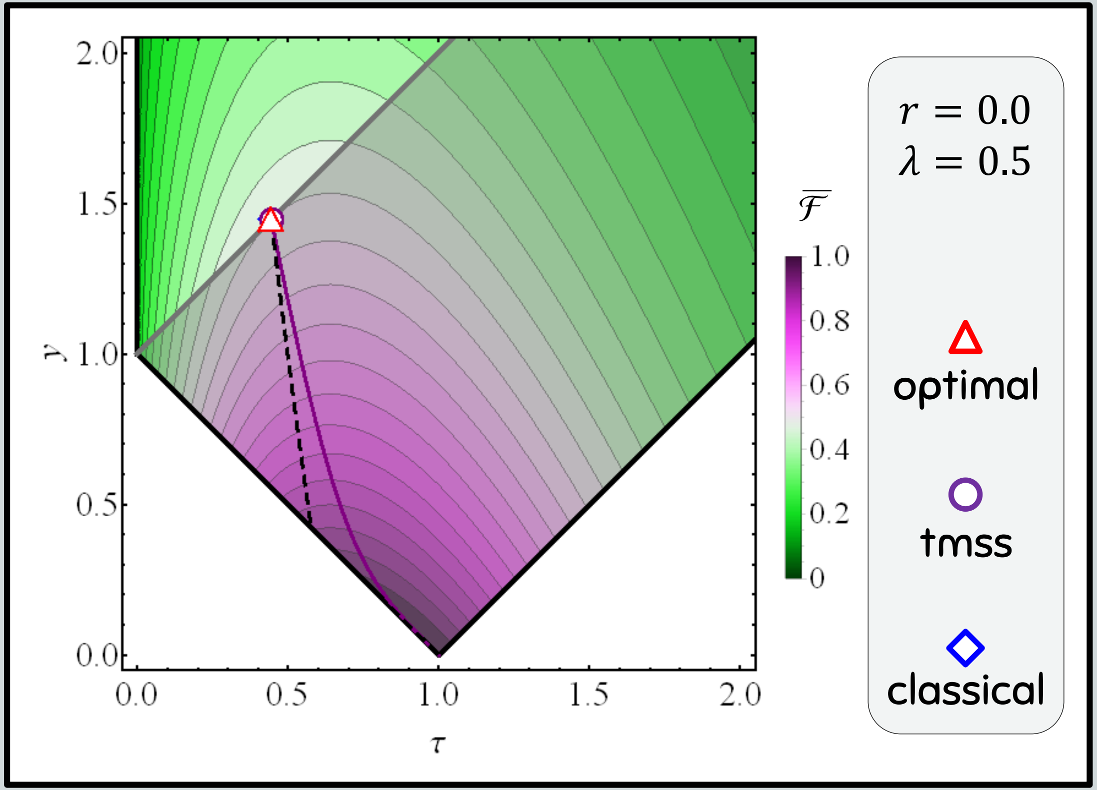
Optimal fidelity: General case

$$\bar{\mathcal{F}}^{\lambda}_{\text{opt}}(r) = \begin{cases} \dfrac{\lambda}{\lambda + \left(1 - \sqrt{\tanh r}\right)^2}, & \tanh r > \dfrac{e^{2r}}{(e^r + \lambda\cosh r)^2} \quad (1) \\ \dfrac{e^r(1+\lambda+\tanh r)}{2e^r + \lambda\cosh r}, & \text{otherwise} \quad (2) \end{cases}$$

- Optimal resource state: mixed asymmetric two-mode Gaussian state with 1 vacuum normal mode [& finite mean energy in case (2)]
- Optimal protocol: BK with non-unit gain depending on r and λ

OPTIMAL AVERAGE FIDELITY

for teleporting an ensemble of input coherent states with variance λ^{-1} using a two-mode Gaussian resource with entanglement $E_N = 2r$



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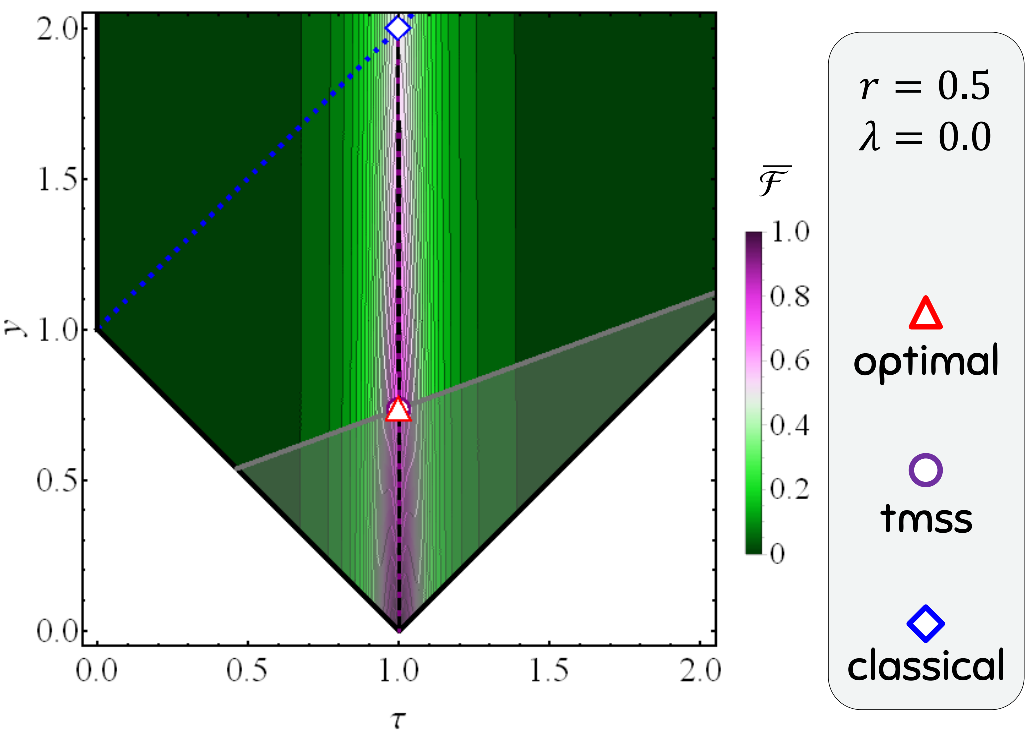

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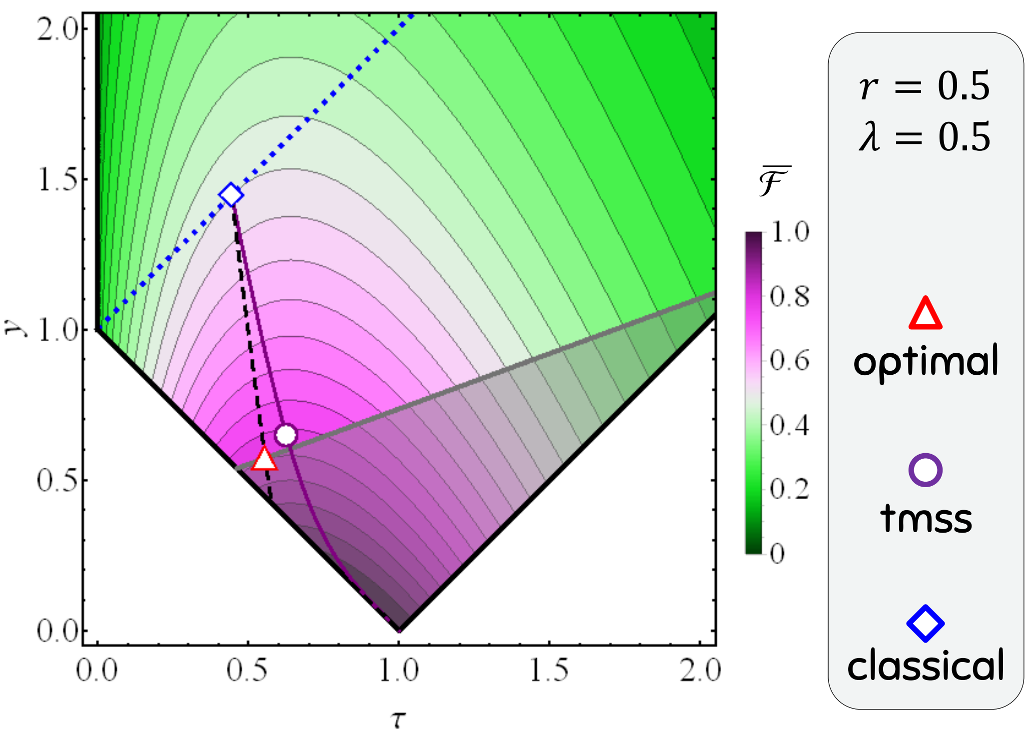
$$E_N = 2r$$



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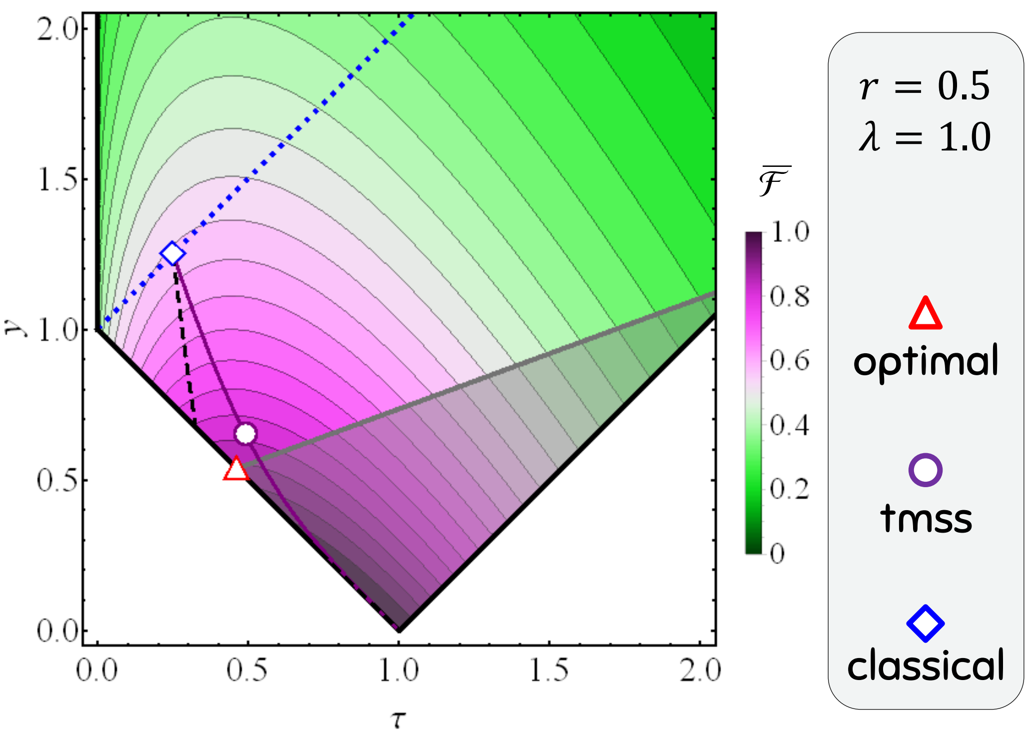
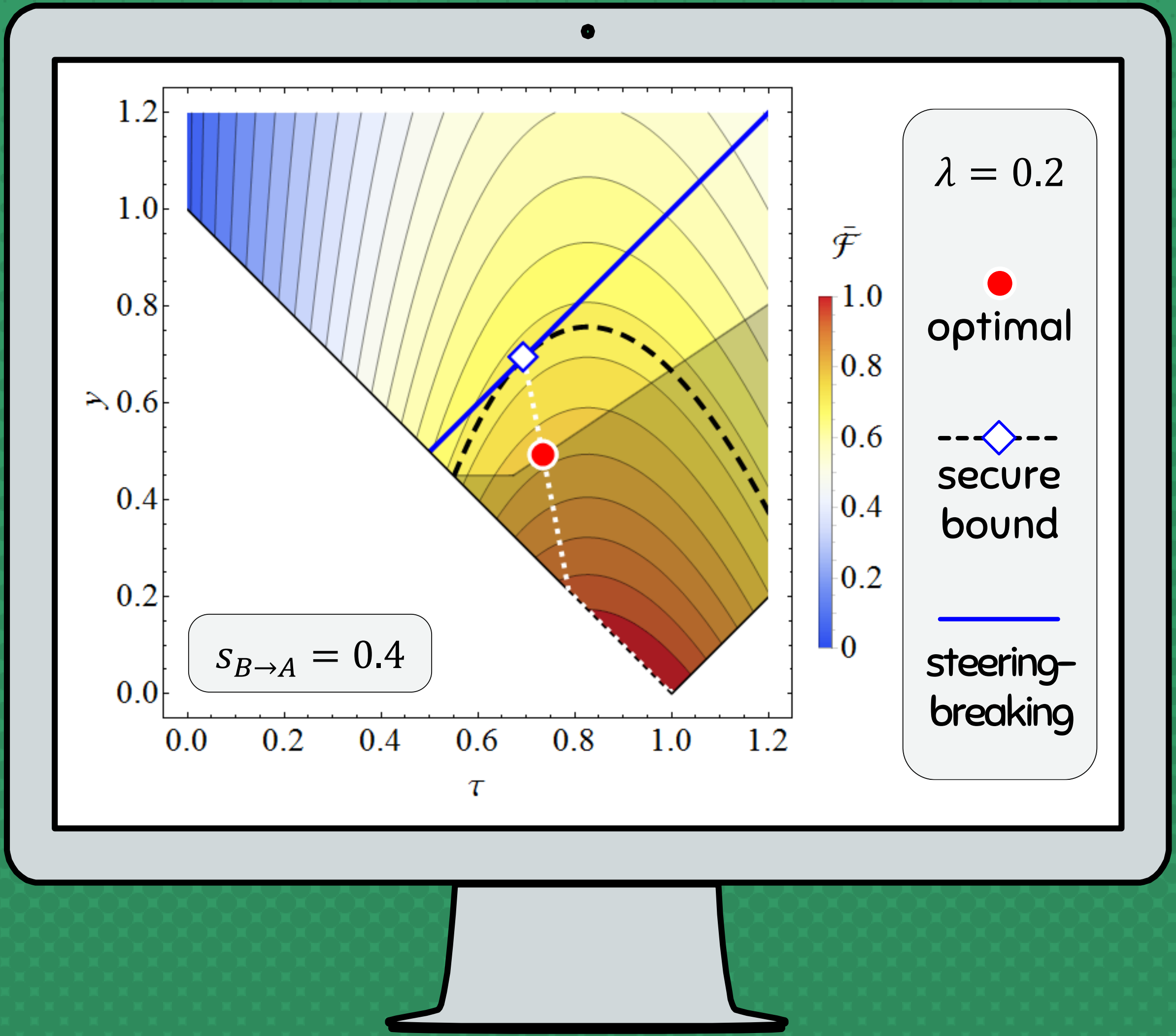


OPTIMAL AVERAGE

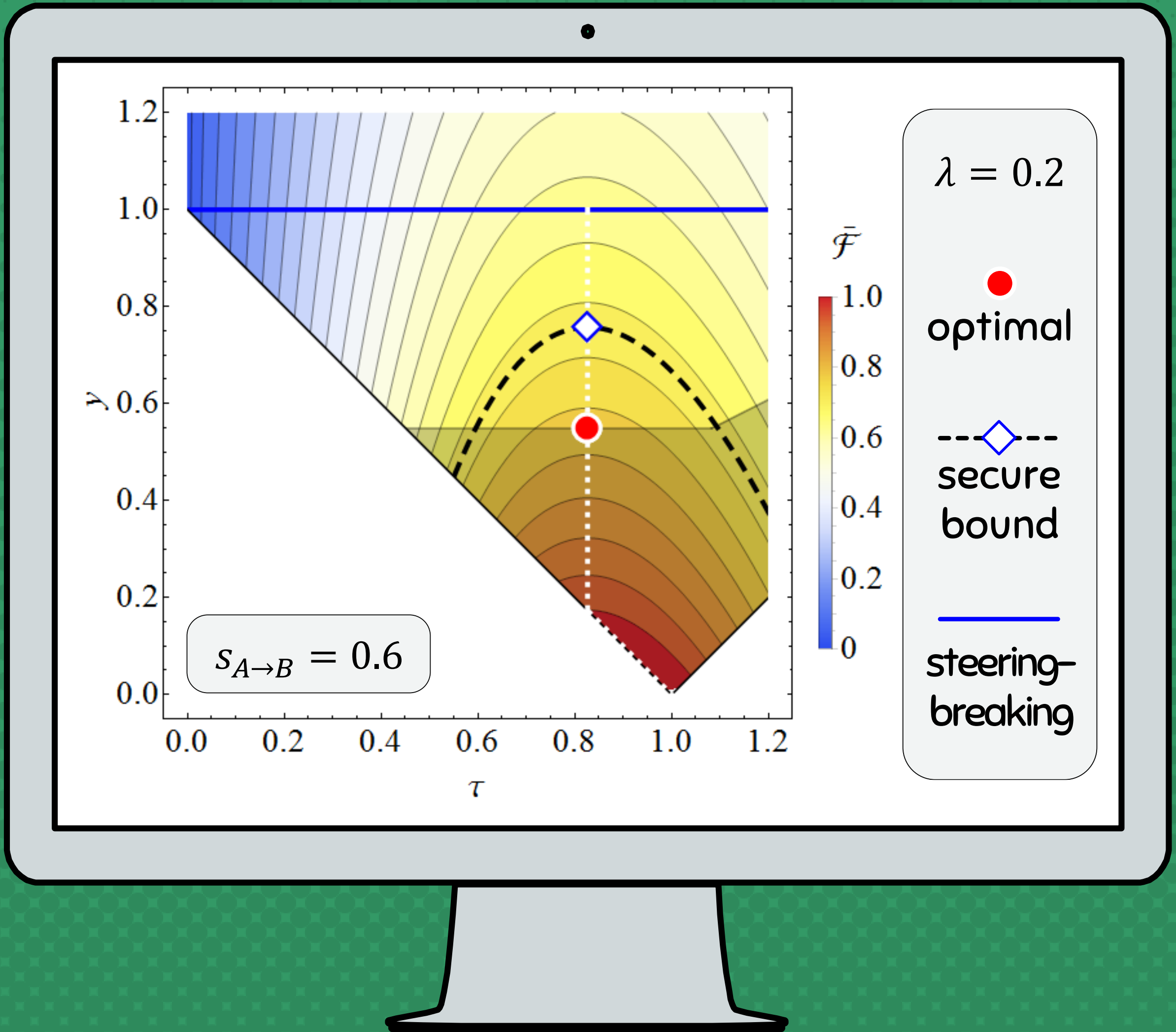
FIDELITY

for teleporting an ensemble of input coherent states with variance λ^{-1} using a two-mode Gaussian resource with fixed steering $s_{B\to A}$ (Bob to Alice)



OPTIMAL AVERAGE FIDELITY

for teleporting an ensemble of input coherent states with variance λ^{-1} using a two-mode Gaussian resource with fixed steering $s_{A\to B}$ (Alice to Bob)



CONCLUSIONS

- Classification of phase-insensitive Gaussian channels implementable by CV teleportation
- Optimal fidelity for teleporting coherent states with fixed entanglement or steering
- Outlook: applications to quantum crypto; use of other resources...

THANKS!

erc

[Physical Review Letters 119, 120503 (2017)]

P. Liuzzo-Scorpo et al, arXiv:1705.03017

[Proceedings SPIE Photonics 10358 (2017)]

P. Liuzzo-Scorpo & GA, arXiv:1708.08548

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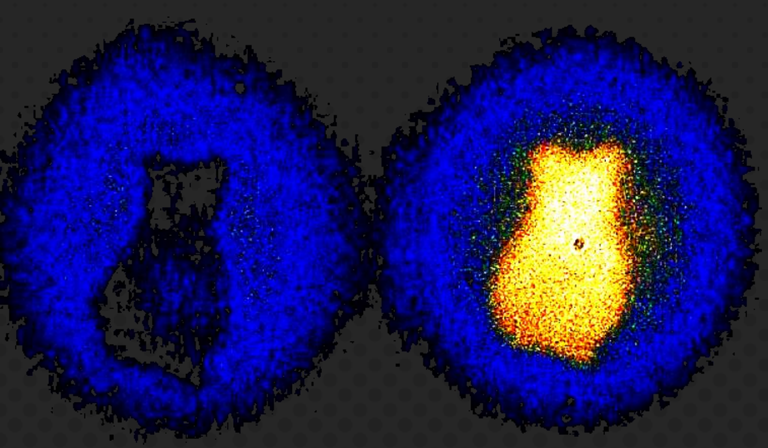
QUANTUM CORRELATIONS GROUP



Foundations of quantum mechanics and their impact on contemporary society

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Image: Destructive and constructive quantum interference obtained by detecting entangled photons that never interacted with the object itself. Credit: Gabriela Barreto Lemos and Victoria Borish. Design: Patricia Enigl. © Austrian Academy of Sciences

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Foundations of quantum mechanics and their impact on contemporary society

Royal Society, London
December 11–12, 2017

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- C. Brukner
- G. Compagno
- G. M. D'Ariano
- L. del Rio
- N. Gisin
- P. Grangier
- C. Rovelli
- B. Terhal
- W. G. Unruh
- J. Vaccaro
- S. Wehner
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- A. Winter
- W. H. Zurek