Characterizing nonclassical correlations via local quantum uncertainty

Davide Girolami — *University of Nottingham*
Tommaso Tufarelli — *Imperial College London*
Gerardo Adesso — *University of Nottingham*

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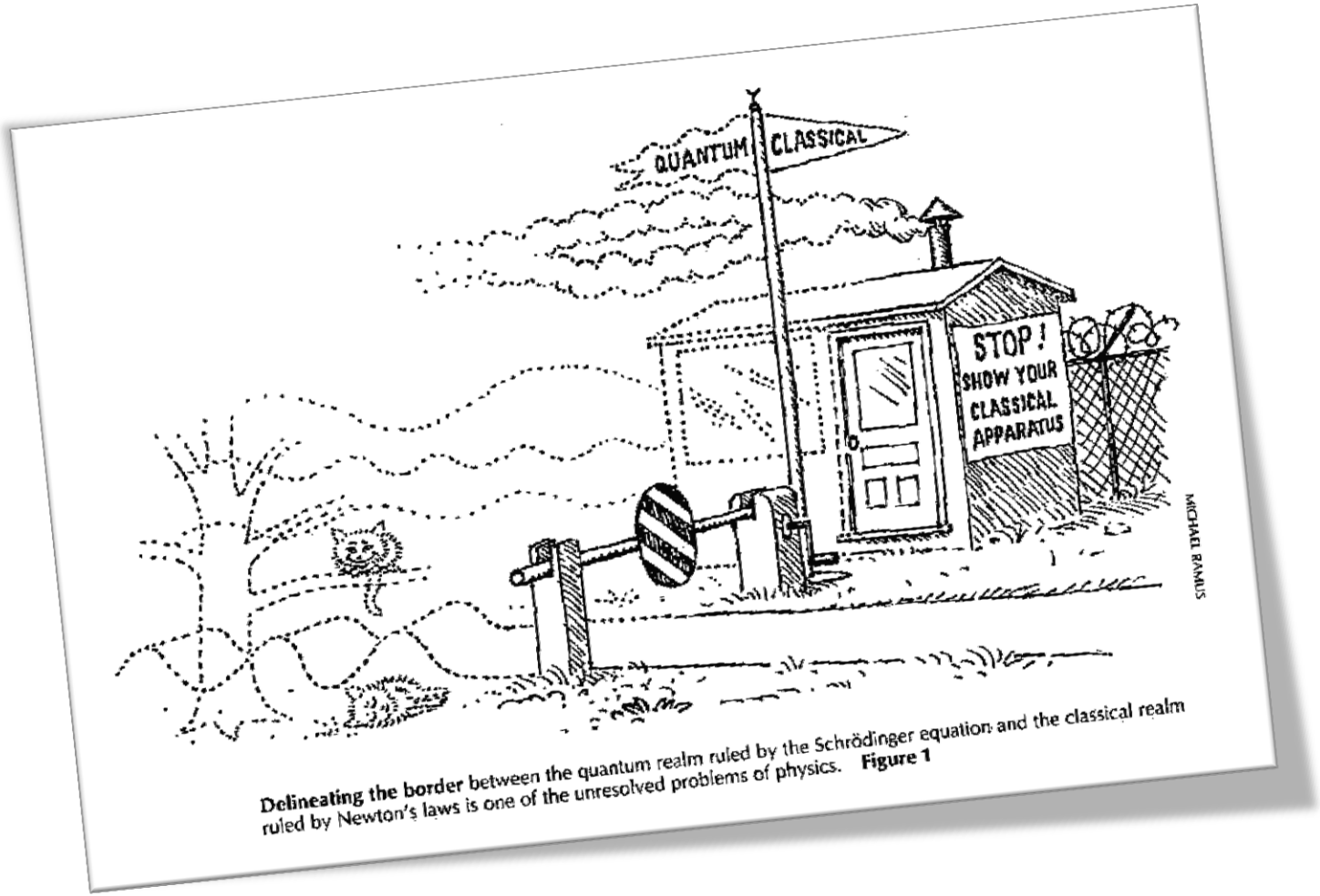
This talk is about…

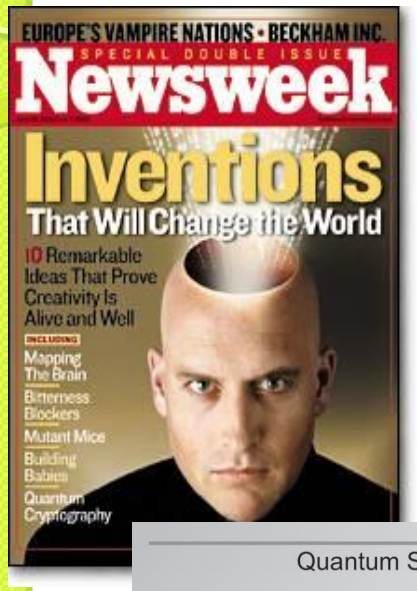


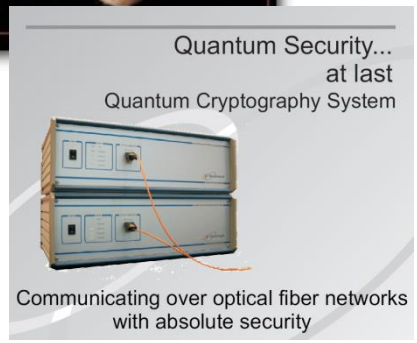
Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Zurek Physics Today 1991

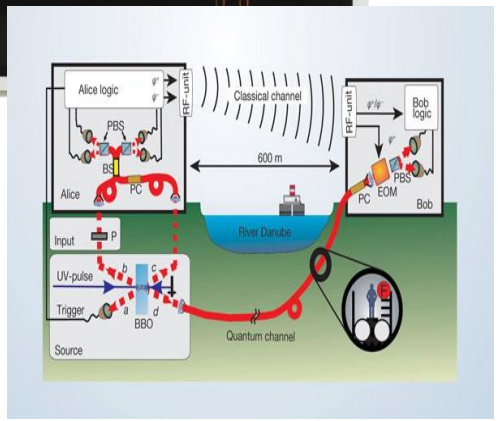
Entanglement

a resource for novel information processing

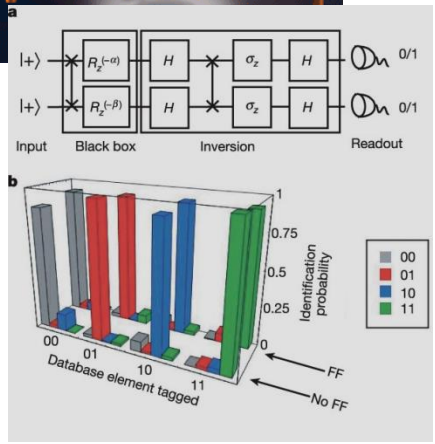




Quantum cryptography



Quantum teleportation



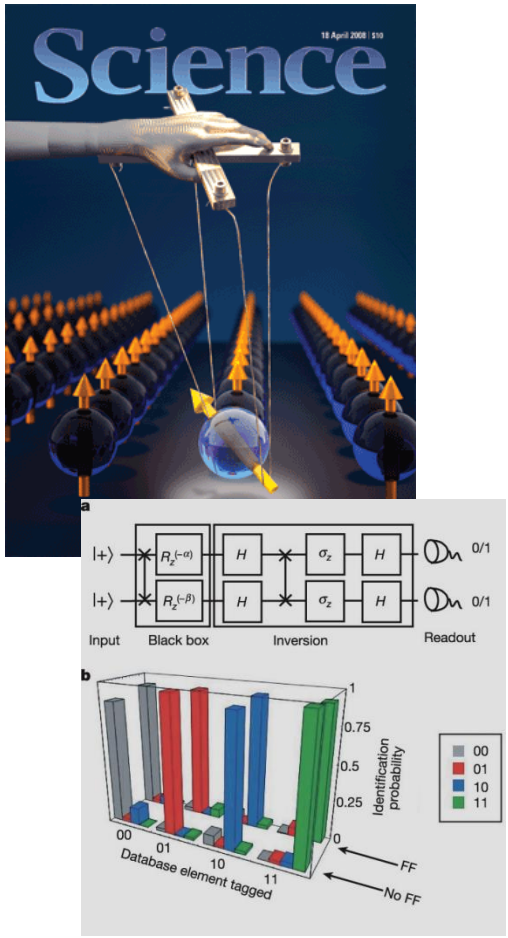
Quantum computation

Quantum correlations

- In certain schemes of quantum computation where the quantum bits are affected by noise, there seems to be a speed-up over classical scenarios even in the presence of negligibly small or vanishing entanglement *(Knill-Laflamme PRL 1998; Datta et al PRL 2008; Barbieri et al PRL 2008)*

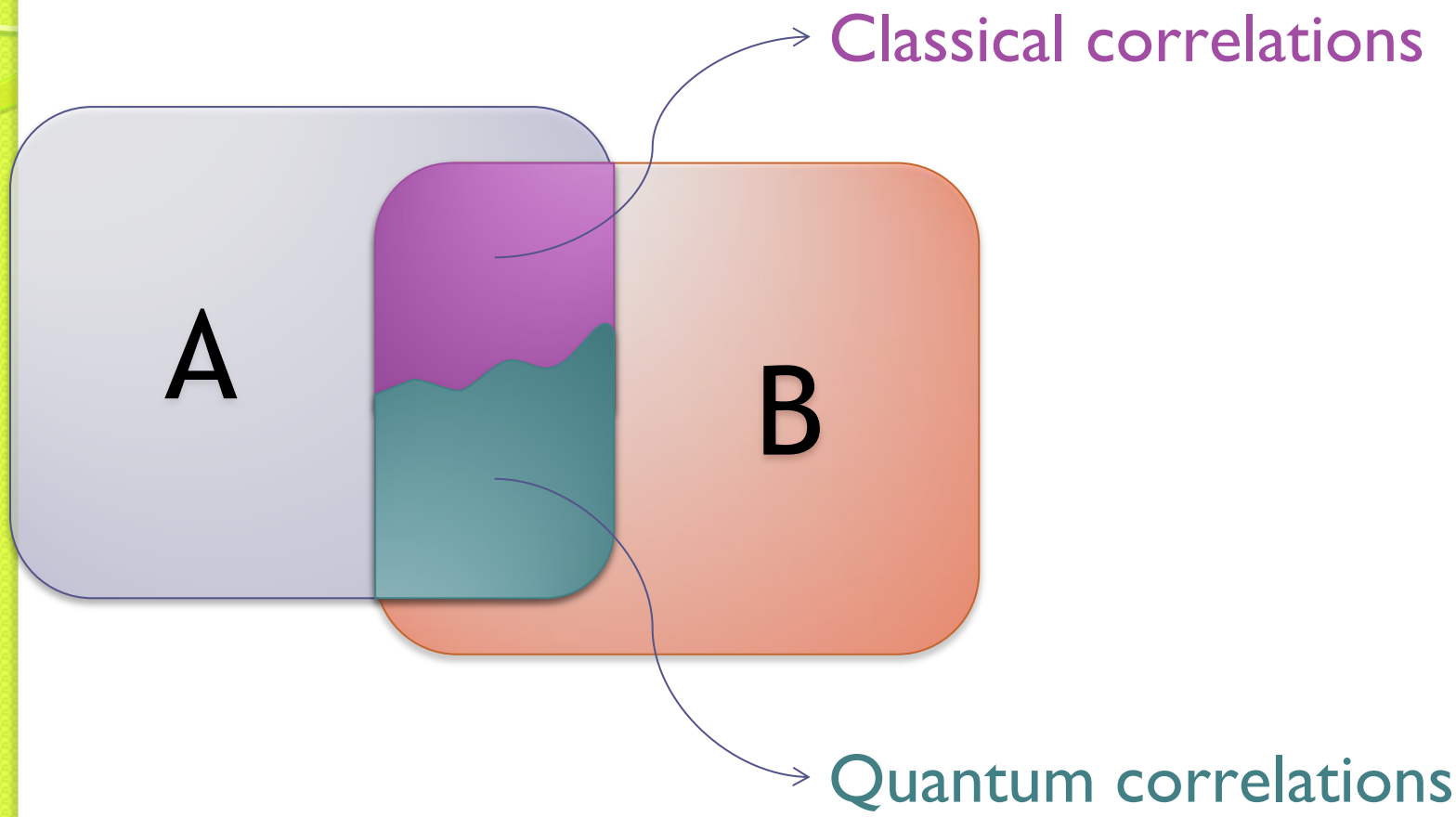
- What is the ultimate resource making quantum processors more effective than classical ones?

- A more refined analysis of the nature of correlations in composite quantum systems is in order

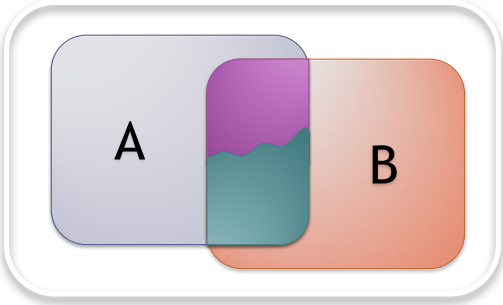


Quantum computation

Correlations

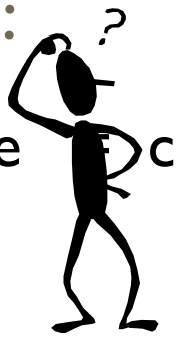


Correlations



- Pure bipartite states:
  - **entanglement** = nonlocality
    nonclassicality (quantum correlations)

- Mixed bipartite states:
  - *Werner 1989*: separable $\Rightarrow$ classically correlated



Quantumness in separable states

Nonorthogonal separable states cannot be discriminated exactly

Measuring a local observable on a separable bipartite state can perturb the state

The eigenvectors of a separable state can be entangled superpositions

...

In general separable states *have not* a purely classical nature

A better paradigm

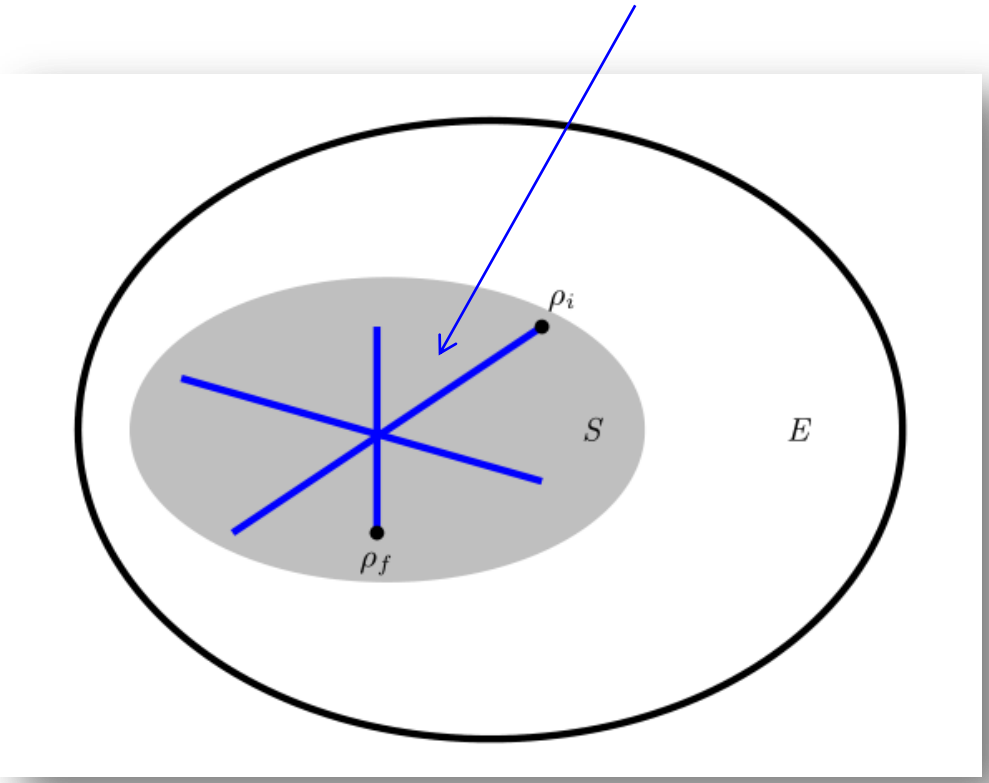
Definition 1. *A bipartite state ρ is: (i)* separable *[20] if it can be written as $\sum_i p_k \sigma_k^A \otimes \sigma_k^B$, where p_k is a probability distribution and each σ_k^X is a quantum state, and* entangled *if non-separable; (ii)* classical-quantum *(CQ) if it can be written as $\sum_i p_i |i\rangle\langle i| \otimes \sigma_i^B$, where $\{|i\rangle\}$ is an orthonormal set, $\{p_i\}$ is a probability distribution and σ_i^B are quantum states; (iii)* classical-classical *(CC), or* (strictly) classically correlated *[3, 5], if there are two orthonormal sets $\{|i\rangle\}$ and $\{|j\rangle\}$ such that $\rho = \sum_{ij} p_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j|$, with p_{ij} a joint probability distribution for the indexes (i, j).*

Groisman et al. arXiv 2007; M. Piani et al. PRL 2008

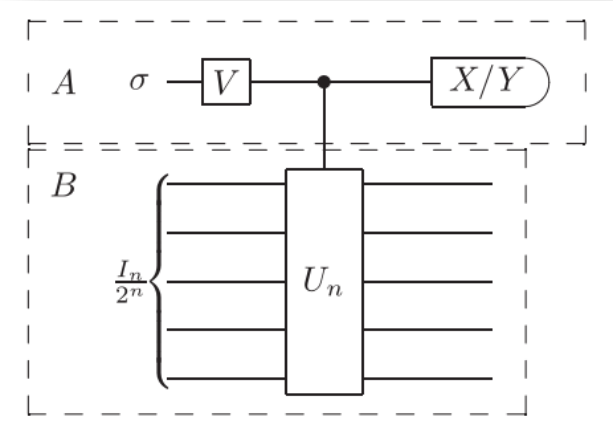
Discordant states

- Almost all bipartite states have non-classical correlations (classical-quantum and strictly classically correlated states are of zero measure)

A. Ferraro et al. PRA 2010

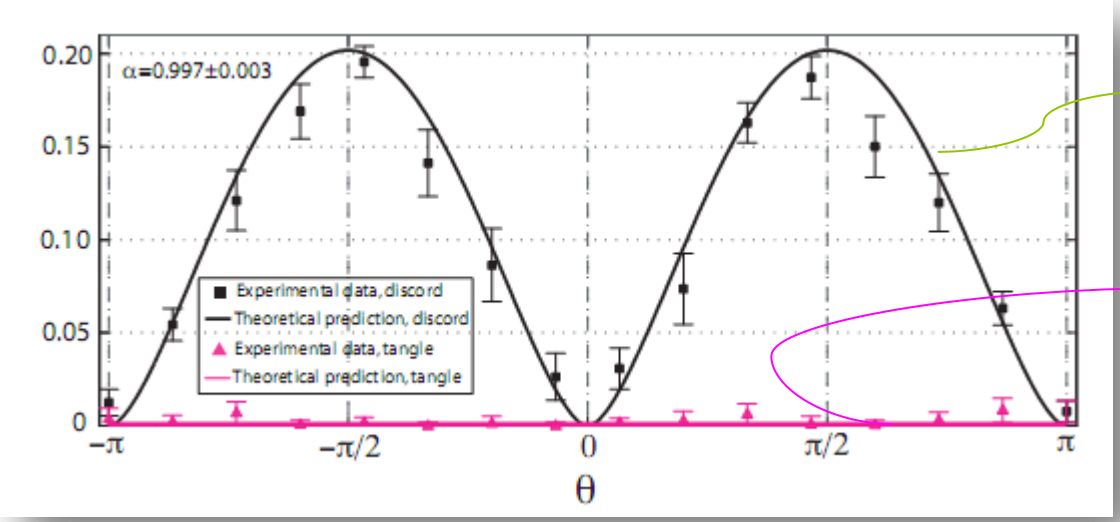


Quantum computation



Non-classical correlations without entanglement might explain the computational speed-up in the DQC1 model of noisy quantum computation

A. Datta et al. 2008-2011;
experiments: M. Barbieri et al. PRL 2008 (photons);
G. Passante et al., PRA 2011 (NMR)



discord

entanglement

Nature June 2011

THE POWER OF DISCORD

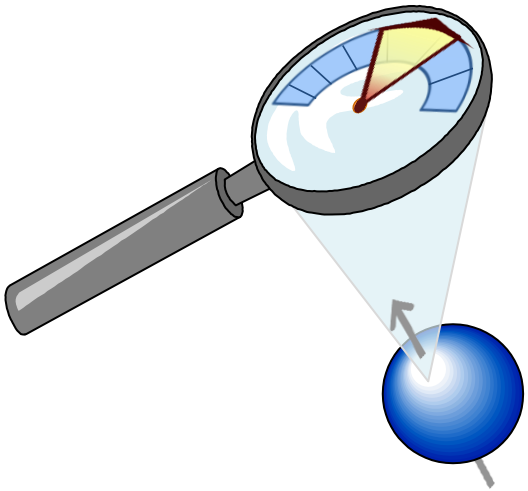
BY ZEEYA MERALI

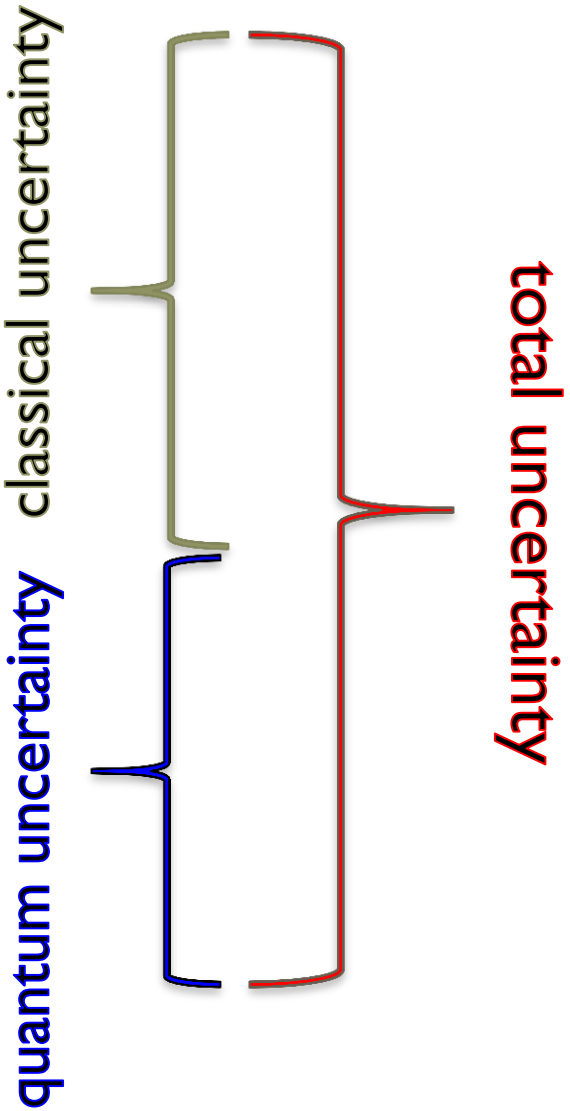
Physicists have always thought quantum computing is hard because quantum states are incredibly fragile. But could noise and messiness actually help things along?

Outline

- Uncertainty on a Single Observable
- Quantum vs Classical Uncertainty
- Quantum vs Classical Correlations
- Local Q Uncertainty = Q Correlations
- Q Correlations-Assisted Metrology

Measurement uncertainty

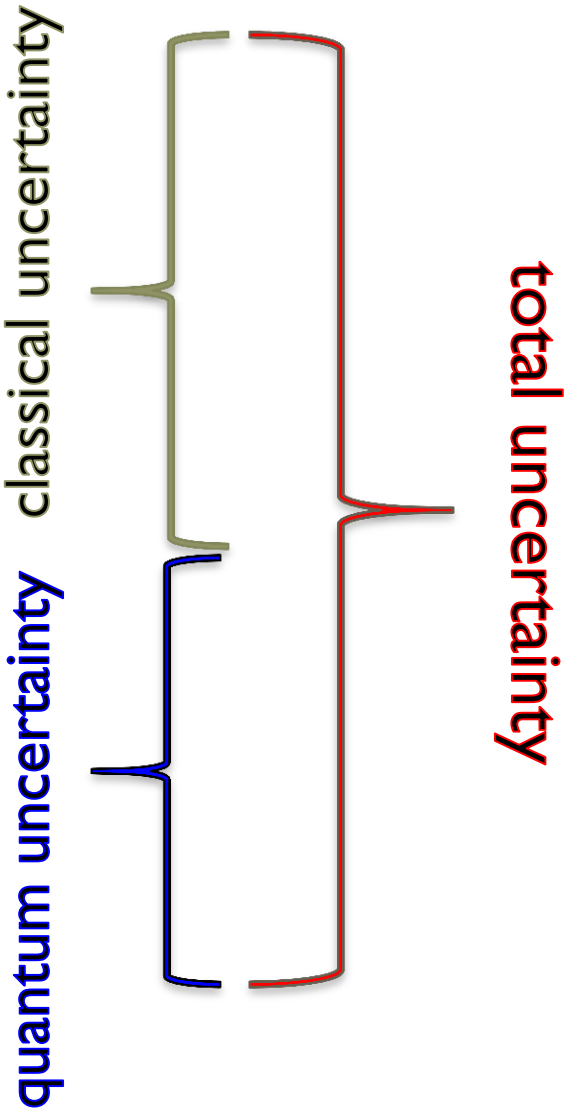




- We have a quantum system in the state ρ
- We measure a single observable K
- Uncertainty = error bar

Measurement uncertainty

- We quantify the total uncertainty (error bar) by the **variance** $\mathrm{Var}_\rho K$
- This may contain **classical ignorance**, due to the state mixedness, and a genuinely **quantum** part, arising due to noncommutativity between state and observable

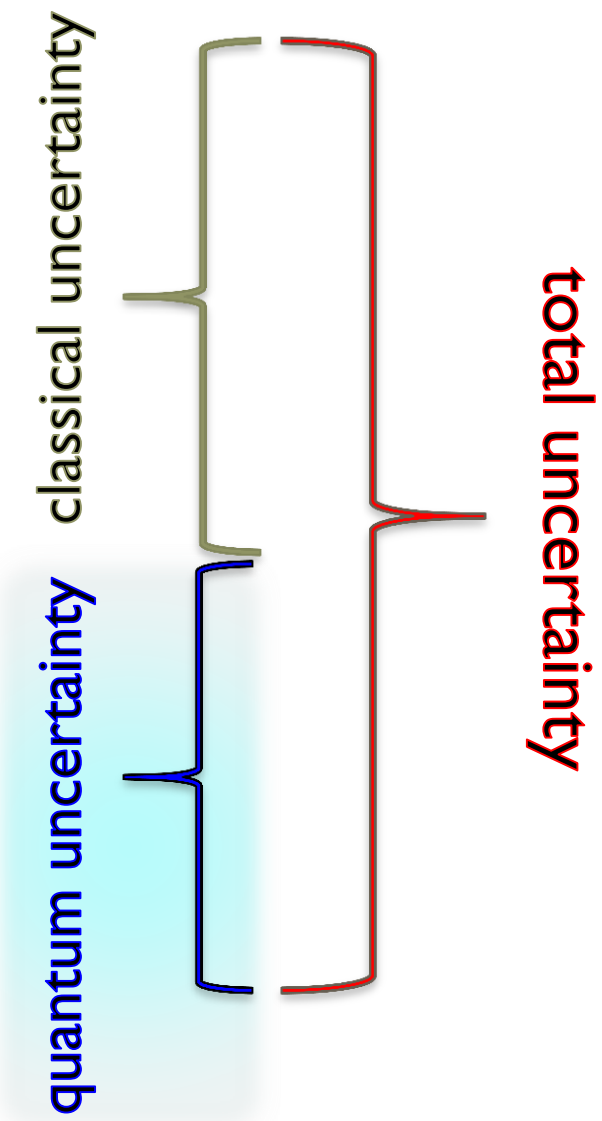


Quantum uncertainty

- We quantify the **quantum uncertainty** on a single observable via the *Wigner-Yanase skew information*

- $\mathcal{I}(\rho, K) = -\frac{1}{2}\mathrm{Tr}\left\{\left[\sqrt{\rho}, K\right]^2\right\}$
$= \mathrm{Tr}[\rho K^2 - \sqrt{\rho}K\sqrt{\rho}K]$

Wigner-Yanase PNAS 1963; Luo PRL 2003



Quantum uncertainty

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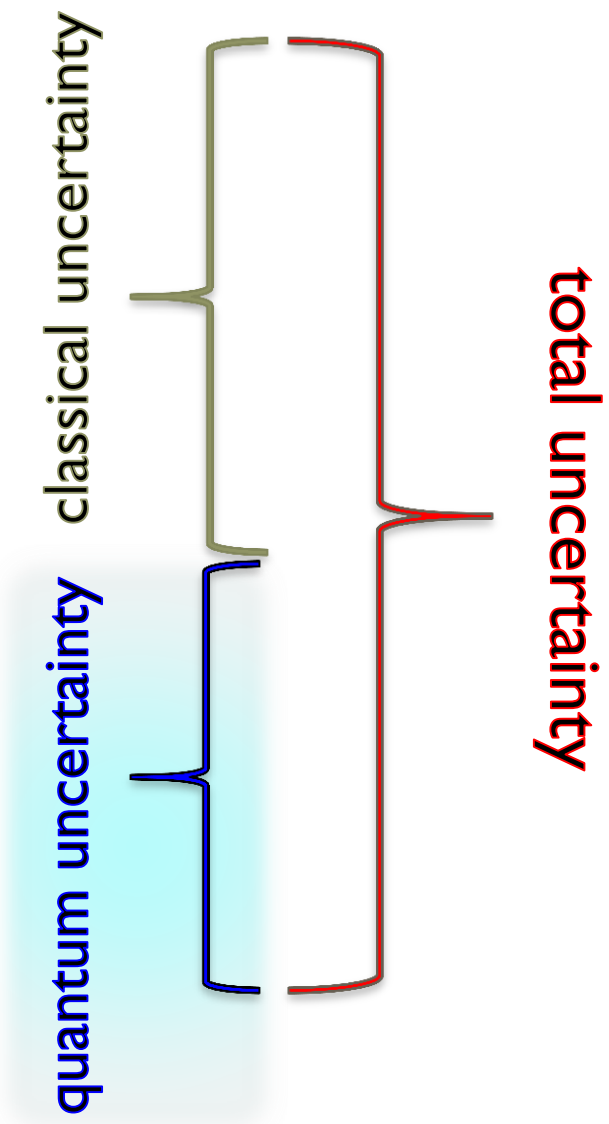
  $= \mathrm{Tr}[\rho K^2 - \sqrt{\rho}K\sqrt{\rho}K]$

- $\mathcal{I}(\rho, K) = 0 \iff [\rho, K] = 0$
- $\mathcal{I}(\rho, K) = \mathrm{Var}_\rho K \iff \rho$ is pure
- $\mathcal{I}(\rho, K)$ is convex
- many other properties…



Quantum uncertainty

Which quantum states unavoidably exhibit quantum uncertainty on the measurement of any single observable?

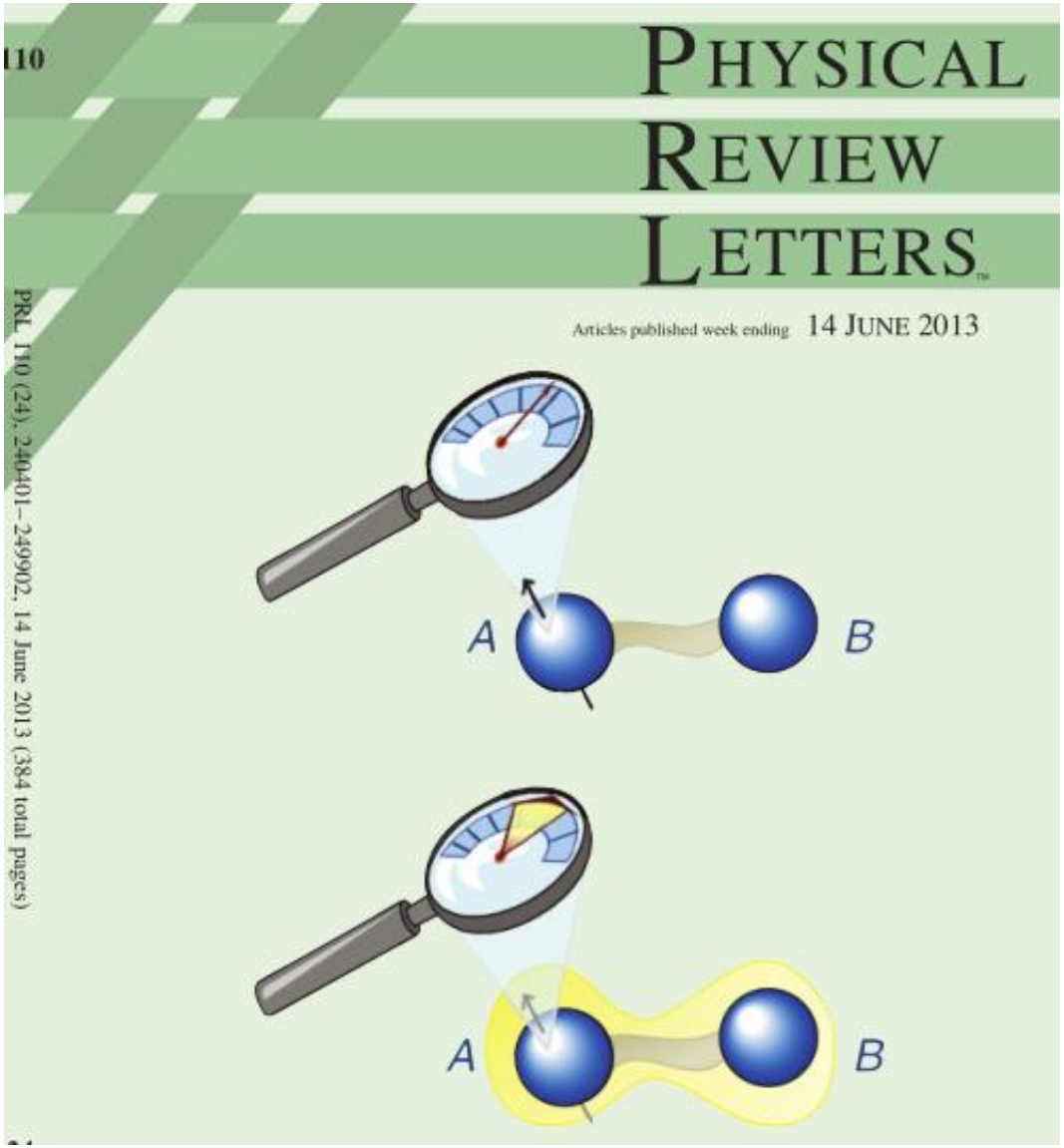
For global observables, **none!**

Quantum uncertainty

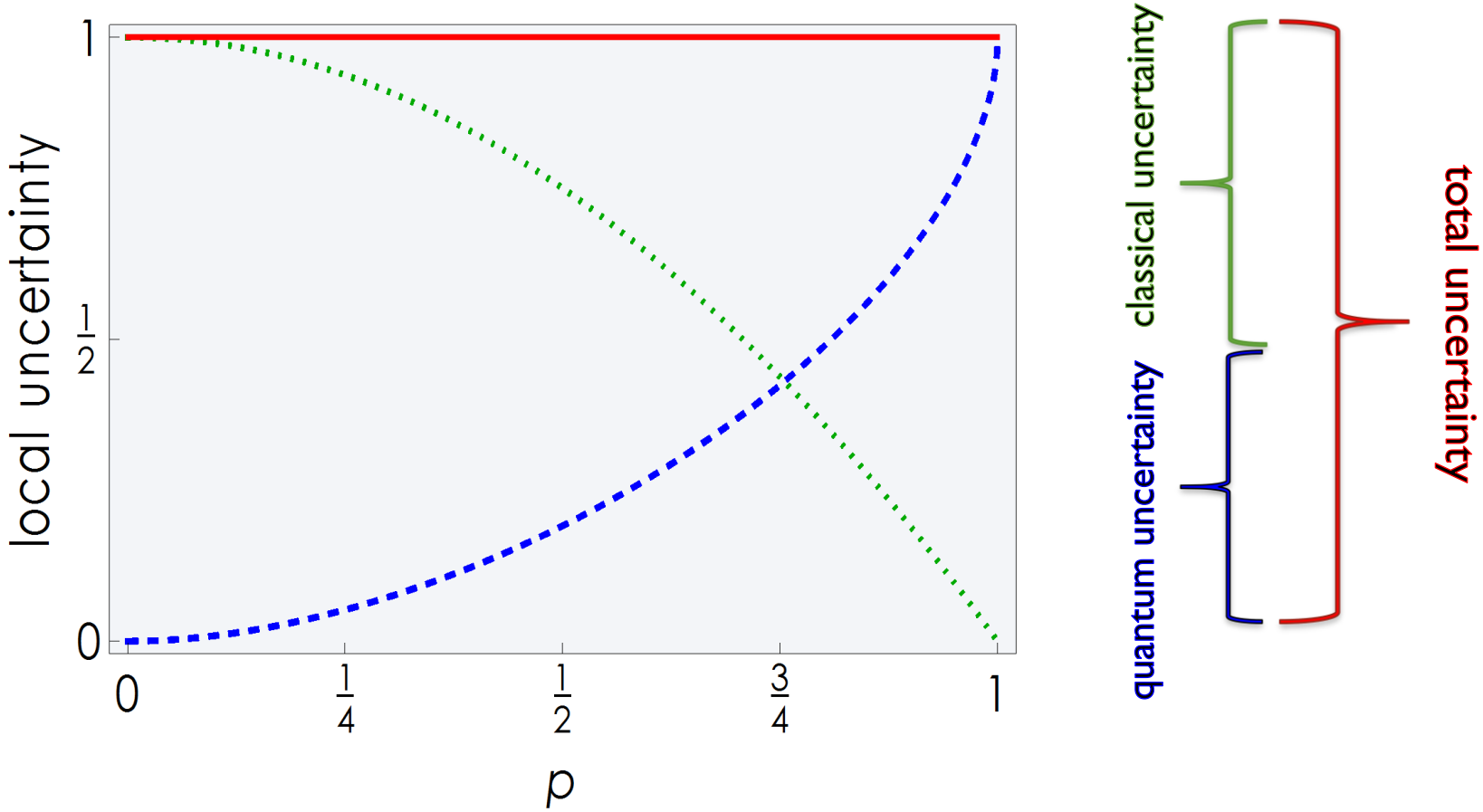
Which quantum states unavoidably exhibit quantum uncertainty on the measurement of any single ***local*** *observable?*

States with nonzero **quantum discord!**

Quantum uncertainty vs discord



Quantum uncertainty vs discord

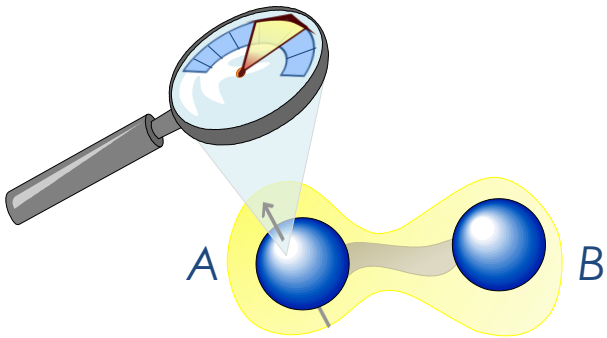


Werner state $\rho_{AB} = p|\phi^+\rangle\langle\phi^+| + \frac{1-p}{4} I_{AB}, \quad K = \sigma_{z\,A} \otimes I_B$

Local quantum uncertainty

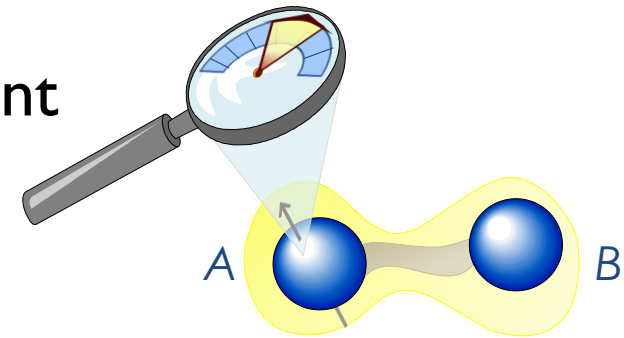
- We focus on a general *bipartite* state $\rho \equiv \rho_{AB}$ and on *local* observables K_A on subsystem A (with nondegenerate spectrum)
- The *minimum* skew information on a single local observable defines the **local quantum uncertainty (LQU):**

$$\mathcal{U}_A(\rho) = \min_{K_A} \mathcal{I}(\rho, K_A)$$



Local quantum uncertainty

- $\mathcal{U}_A(\rho\,) = \min_{K_A} \mathcal{I}(\rho, K_A)$
- The LQU is a *bona fide* measure of discord
  - It is invariant under local unitaries and nonincreasing under quantum operations on *B*
  - It vanishes if and only if ρ is classical-quantum, $\rho = \sum_i p_i |i\rangle\langle i|_A \otimes \tau_{i\,B}$
  - It reduces to an entanglement monotone for pure states

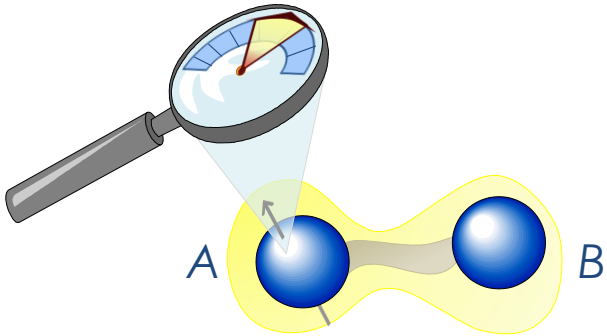


Local quantum uncertainty

- $\mathcal{U}_A(\rho) = \min_{K_A} \mathcal{I}(\rho, K_A)$
- The LQU is a *computable* measure of discord for 2x*d* systems
- $\boxed{\mathcal{U}_A(\rho_{AB}) = 1 - \lambda_{\max}(W_{AB})}$

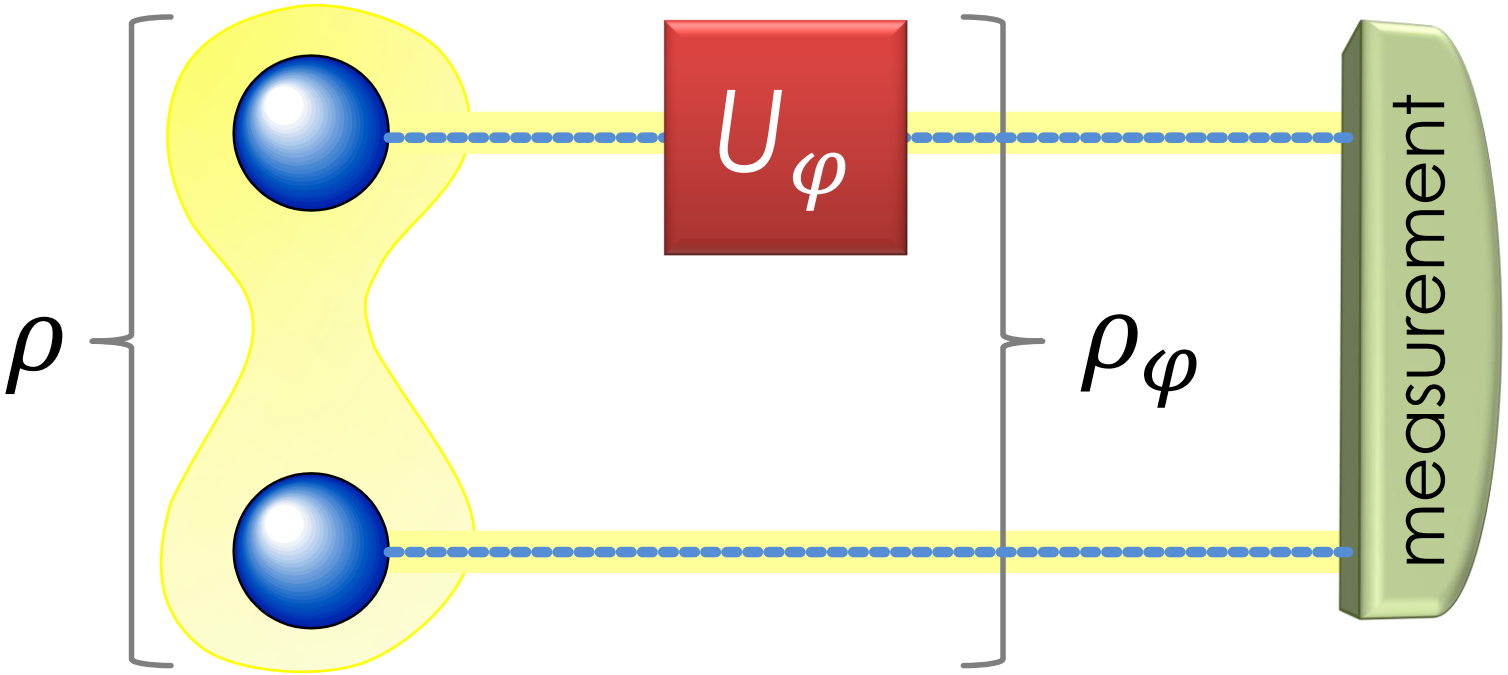
$$(W_{AB})_{ij} = \mathrm{Tr}\left\{\rho_{AB}^{1/2}\left(\sigma_{i\,A} \otimes \mathbb{I}_B\right)\rho_{AB}^{1/2}\left(\sigma_{j\,A} \otimes \mathbb{I}_B\right)\right\},$$
with $i, j = x, y, z$.

- geometric interpretation (Hellinger distance)



Applications to metrology

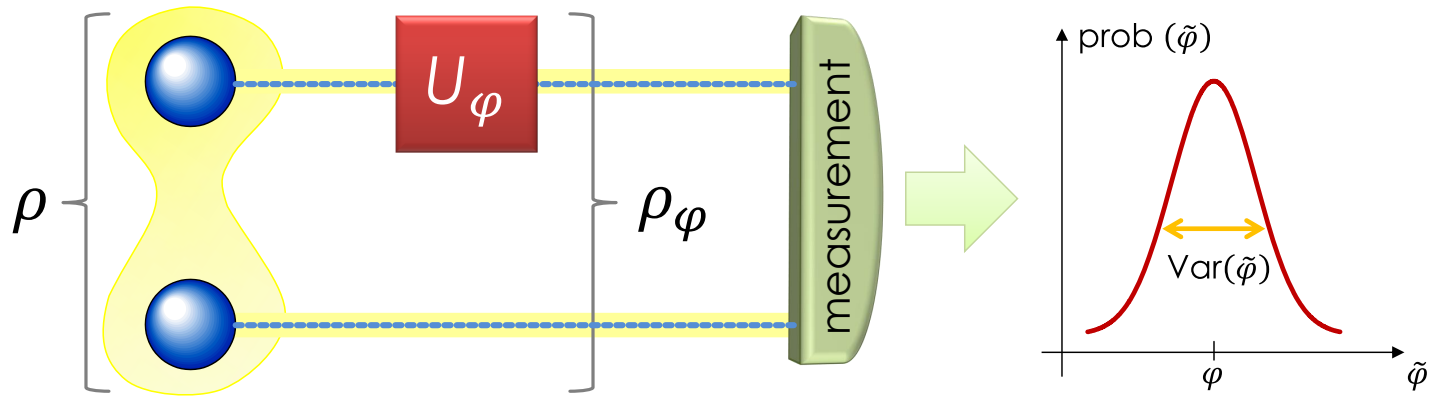
- Parameter estimation with mixed probes



$U_\varphi = e^{-i\varphi H_A}$, where H_A is a nontrivial local Hamiltonian

Applications to metrology

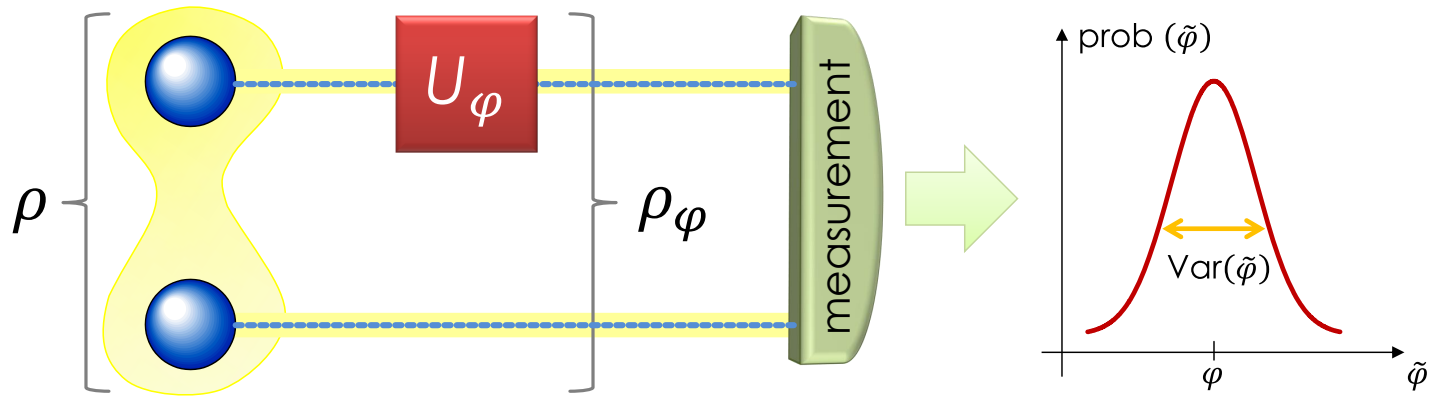
- Parameter estimation with mixed probes



- For $\nu \gg 1$ repetitions of the experiment, the best estimator saturates the Cramer-Rao bound
- $\mathrm{Var}(\tilde{\varphi}_{best}) = \frac{1}{\nu \mathcal{F}(\rho_\varphi)}$
- $\mathcal{F}(\rho_\varphi)$ quantum Fisher information

Applications to metrology

- Parameter estimation with mixed probes



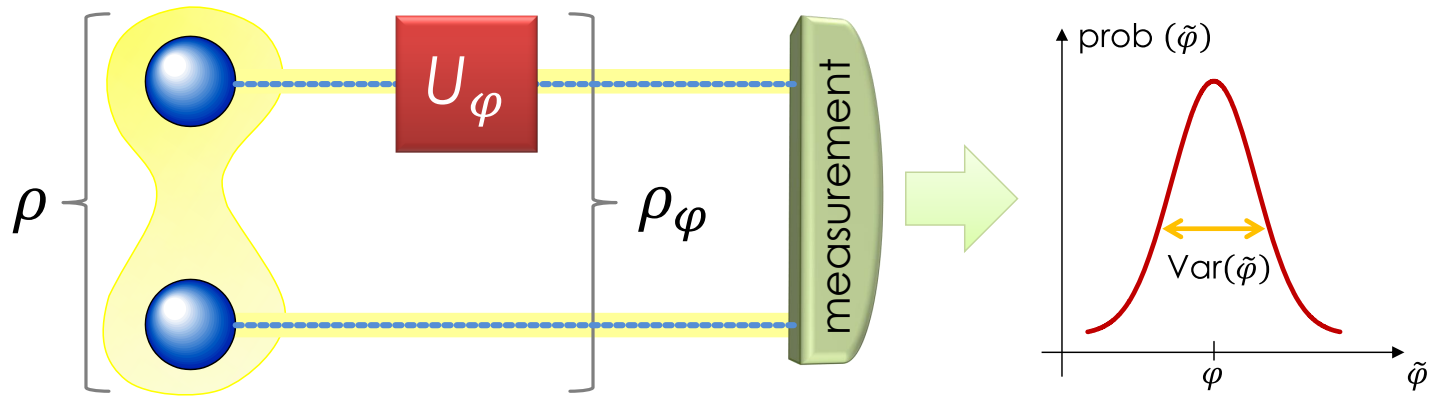
- $\mathcal{U}_A(\rho) \leq \mathcal{I}(\rho, H_A) = \mathcal{I}(\rho_\varphi, H_A) \leq \frac{1}{4}\mathcal{F}(\rho_\phi)$

for $\nu \gg 1$ repetitions and for the optimal estimator

$$\mathrm{Var}(\tilde{\varphi}_{best}) \leq \frac{1}{4\nu\, \mathcal{U}_A(\rho)}$$

Applications to metrology

- Parameter estimation with mixed probes



- **Discord measured by LQU sets *a guaranteed upper bound* on the variance of phase estimation with mixed probes and optimal measurements**

$$\mathrm{Var}(\tilde{\varphi}_{best}) \leq \frac{1}{4\nu\, \mathcal{U}_A(\rho)}$$

Conclusions

- Quantum states can display quantum uncertainty on a single observable, quantifiable via the Wigner-Yanase skew information
- For a bipartite system, this is unavoidable as soon as there are discord-type nonclassical correlations
- The minimum quantum uncertainty on a local observable defines a *bona fide* and computable measure of discord
- Quantum discord in the mixed probe states used for phase estimation ensures that the optimal metrological scheme returns an estimator with variance bounded from above by the inverse of discord.

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Thank you!