QUANTUM BENCHMARKS FOR GAUSSIAN STATES

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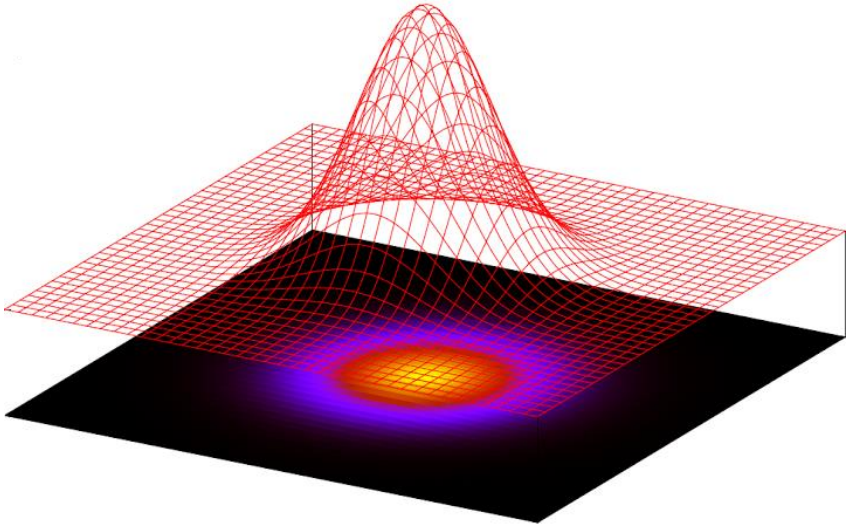

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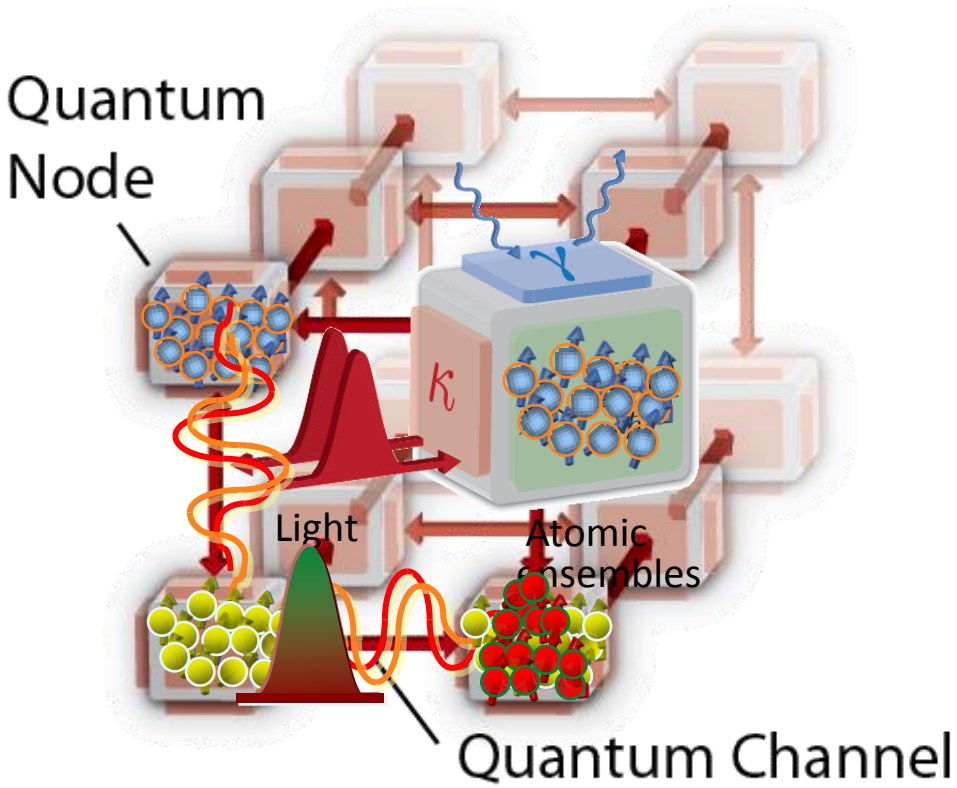
The quantum internet

H. J. Kimble[1]

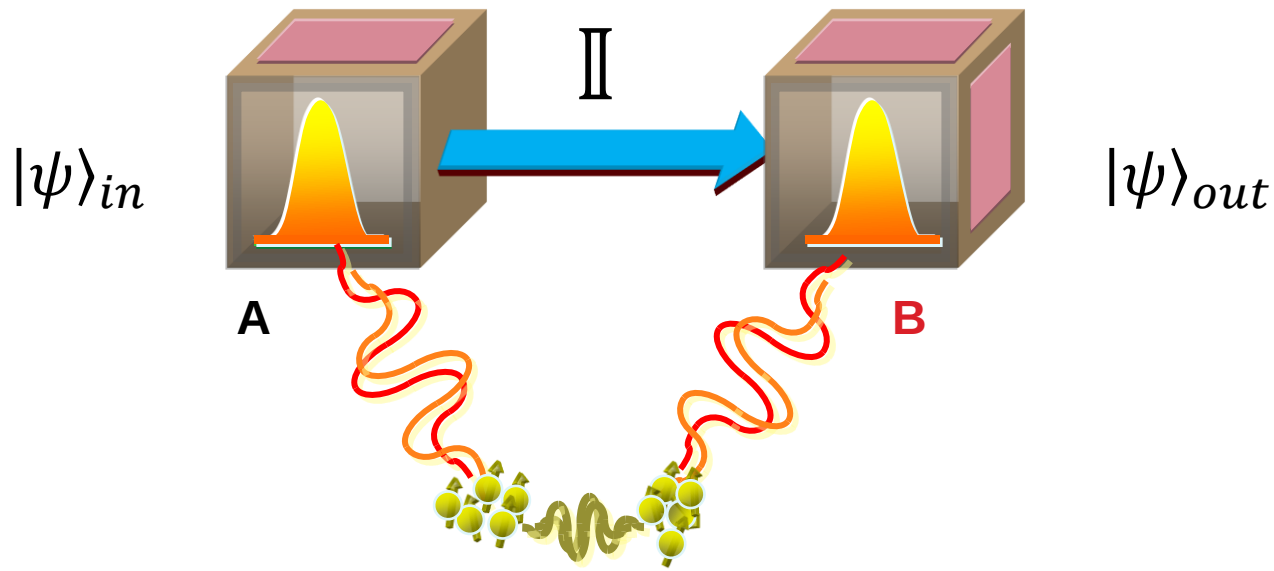


In order to implement **quantum interfaces** one needs to be able to:

- ✓ ***Entangle*** multiple nodes
- ✓ ***Teleport*** information through the channels
- ✓ ***Store and retrieve*** quantum states from light to matter

Quantum teleportation and storage

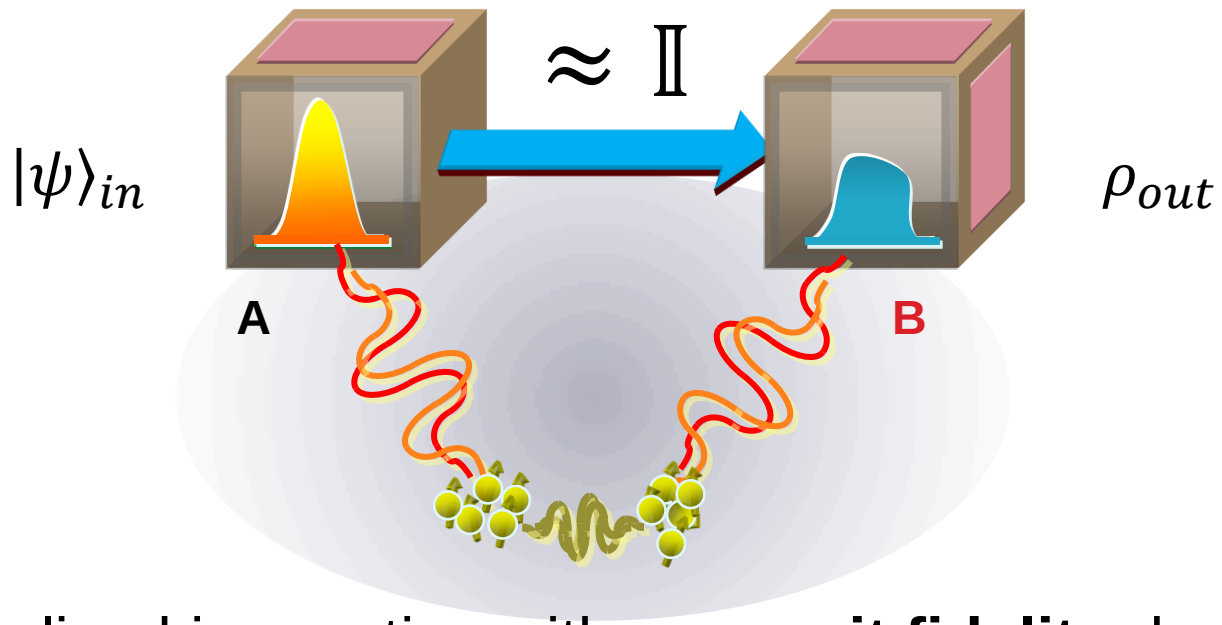
- are fundamental ingredients of quantum networks
- are implementations of the **identity** channel



- can be perfectly realised in principle by sharing maximally entangled states (assisted by classical communication)

Quantum teleportation and storage

- are fundamental ingredients of quantum networks
- are implementations of the **identity** channel



- are realised in practice with **non-unit fidelity** due to various sources of imperfections, noise, etc.

Quantum teleportation and storage

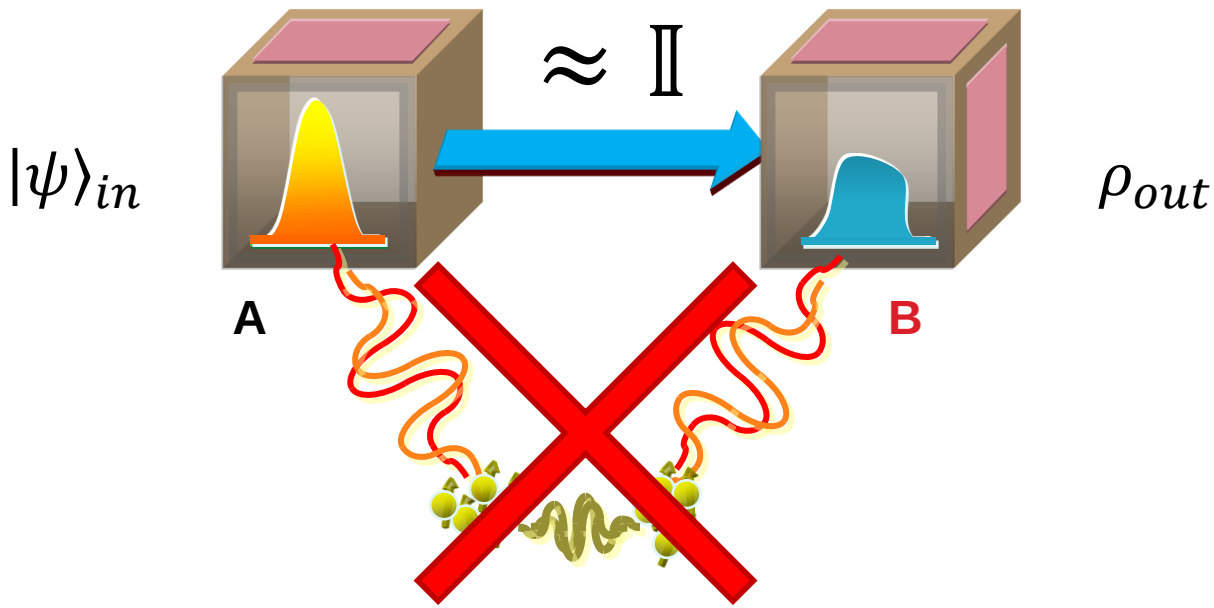
$|\psi\rangle_{in}$

ρ_{out}

- are realised in practice with **non-unit fidelity** due to various sources of imperfections, noise, etc.

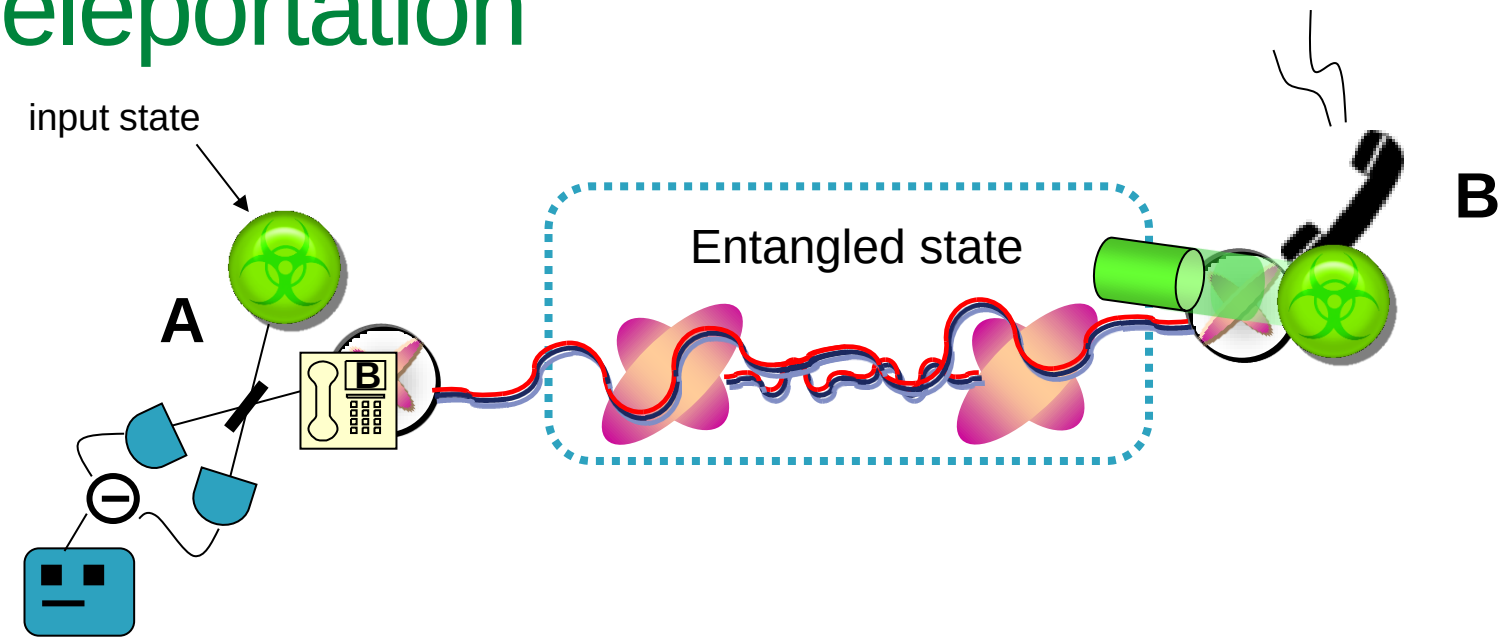
Quantum benchmarks

- set thresholds for the best approximation of the identity channel (or other channels) without quantum resources



Quality control for quantum technology

Teleportation



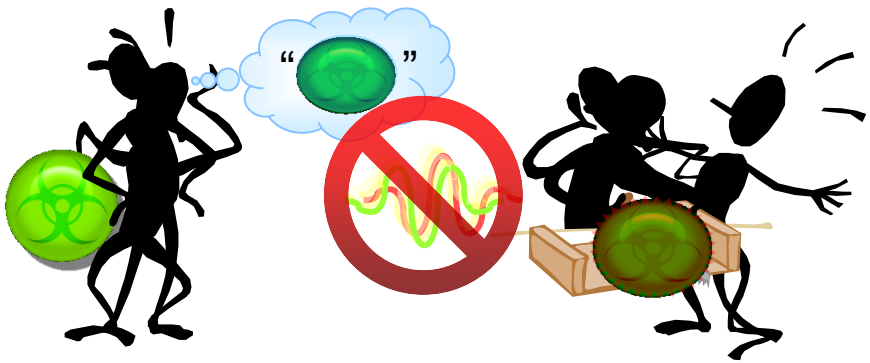
Fidelity

$|\psi_{in}\rangle$

ρ_{out}^{Q}

$$F_Q = \langle \psi_{in} | \rho_{out}^{Q} | \psi_{in} \rangle$$

“Classical” strategy



Measure & Prepare

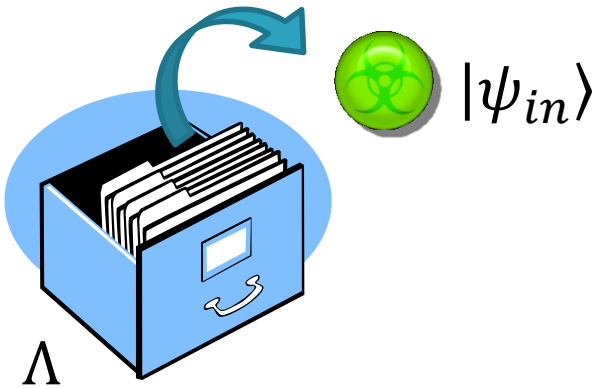
Fidelity

 $|\psi_{in}\rangle$  $\rho_{out}^{M\&P}$

$$F_{M\&P} = \langle\psi_{in}|\rho_{out}^{M\&P}|\psi_{in}\rangle$$

Formalising the problem

- The input states are drawn from an input set $\Lambda = \{|\psi_{in}\rangle, p_\psi\}$ with a certain prior probability distribution p_ψ.
- The specifics of the input set are **known** to Alice and Bob



- Average fidelities $\bar{F}$ are calculated (for classical or quantum strategies, respectively) by averaging over the input set

$$\bar{F} = \sum_{\psi \in \Lambda} p_\psi \langle \psi_{in} | \rho_{out} | \psi_{in} \rangle$$

Classical fidelity threshold (CFT)

- Best possible average fidelity without entanglement

$$\bar{F}_C = \sup_{M\&P} \bar{F}_{M\&P} = \sup_{M\&P} \left(\sum_{\psi \in \Lambda} p_\psi \langle \psi_{in} | \rho_{out}^{M\&P} | \psi_{in} \rangle \right)$$

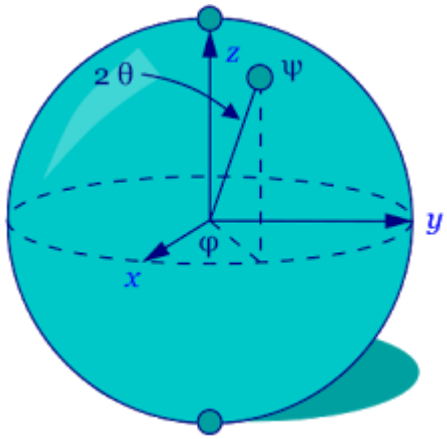
- The CFT sets a **Quantum Benchmark** for transmission of states drawn from the set Λ

- Quantum implementations need to achieve

$$\bar{F}_Q > \bar{F}_C$$

to certify that entanglement has been used

Quantum benchmarks: known results



Input: **qudit states**

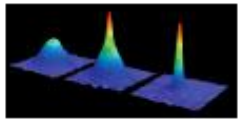
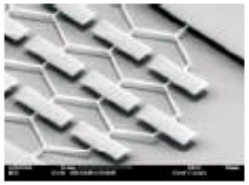
randomly sampled from $\mathbb{C}^d$

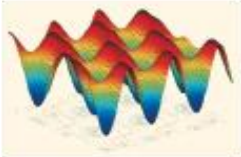
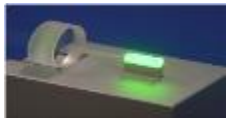
Benchmark: $\bar{F}_C = \frac{2}{d+1}$

(Bruss-Macchiavello, PLA 1999)

- Continuous variable limit: $\bar{F}_C \xrightarrow[d\to\infty]{} 0$
- Realistic experiments deal with a restricted class of states
  - Typically, **Gaussian states**

Gaussian states

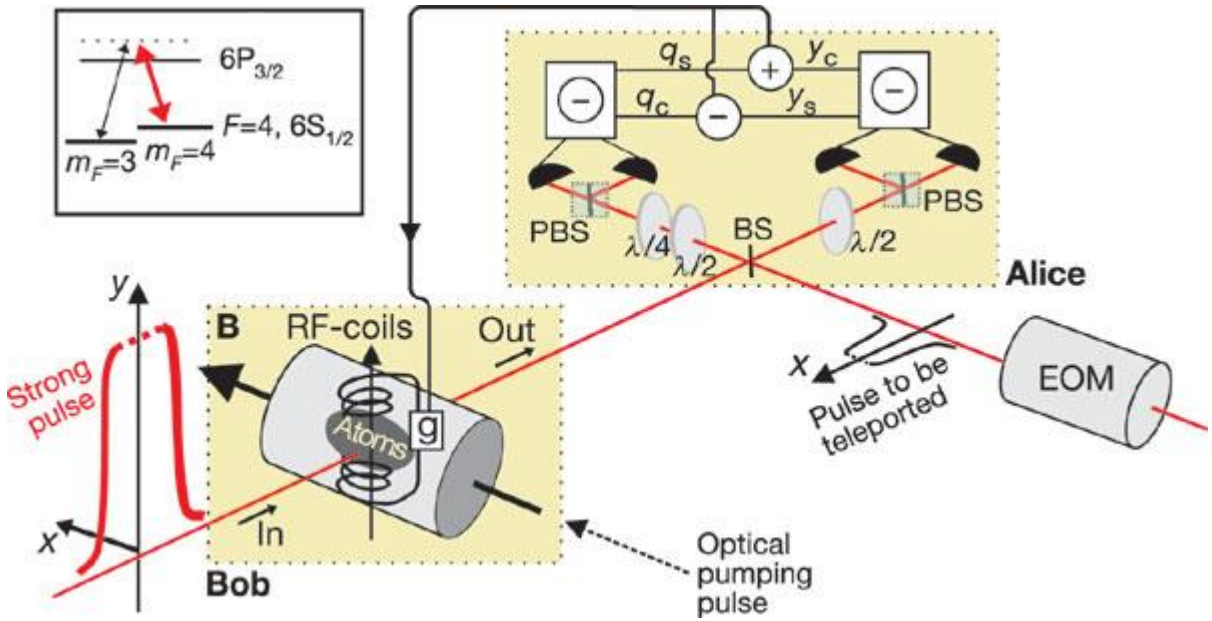



- Very natural: ground and thermal states of all physical systems in the harmonic approximation regime
- Relevant *theoretical* testbeds for the study of structural properties of entanglement and correlations, thanks to the symplectic formalism
- Preferred resources for *experimental* unconditional implementations of continuous variable protocols
- Crucial role and remarkable control in quantum optics
  - coherent states
  - squeezed states
  - thermal states

Gaussian states teleportation/storage

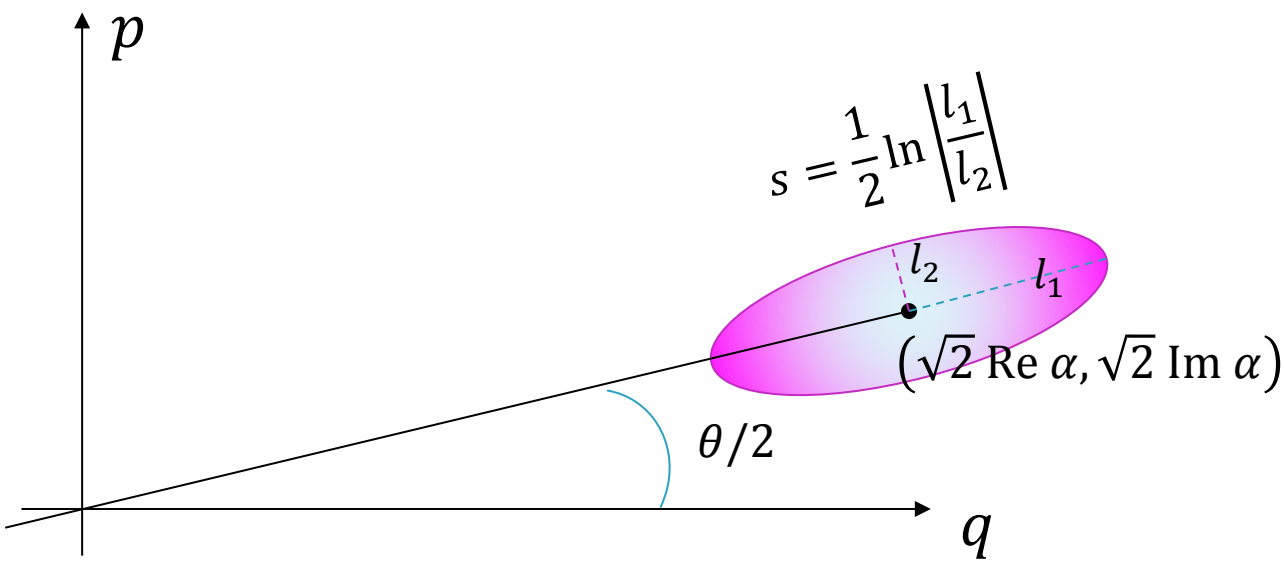
- Quantum memory for light (coherent states, squeezed states) onto atoms *(Nature 2004, Nature Phys 2011)*
- Quantum teleportation between light and matter *(Nature 2006)*



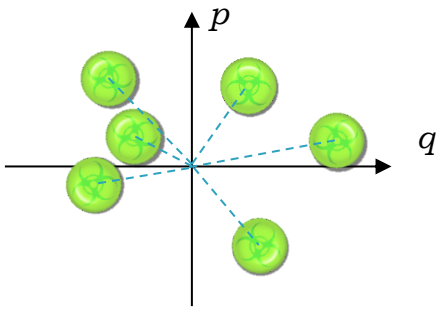
Pure single-mode Gaussian states

$$|\psi\rangle = D(\alpha)S(\xi)|0\rangle$$

- Displacement operator: $D(\alpha) = \exp\left(\alpha a^\dagger - \alpha^* a\right)$
- Squeezing operator: $S\left(\xi \equiv s\, \mathrm{e}^{i\theta}\right) = \exp\left(\frac{1}{2}\xi a^{\dagger 2} - \xi^* a^2\right)$



Gaussian benchmarks: special cases



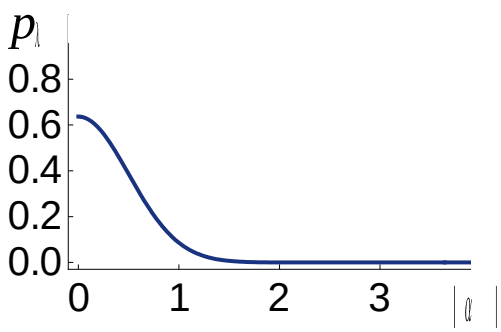
Input: **coherent states** ($\xi = 0$)

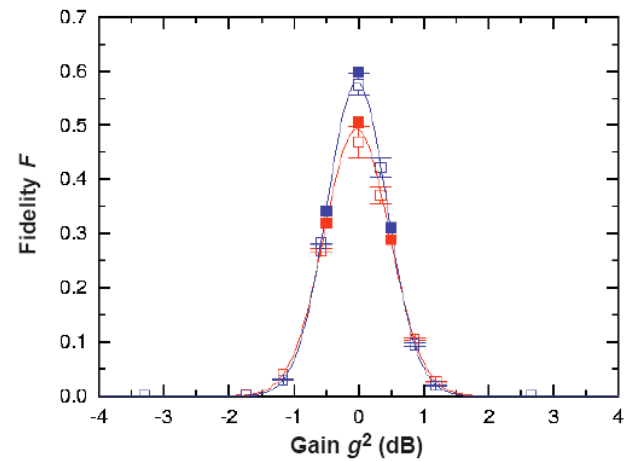
with Gaussian displacement distribution, inverse width λ

Benchmark: $$\bar{F}_C^{coh}(\lambda) = \frac{1+\lambda}{2+\lambda} \xrightarrow[\lambda \to 0]{} \frac{1}{2}$$

(Hammerer et al. PRL 2005)

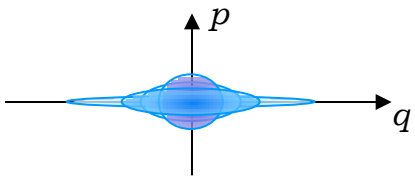
$$p_\lambda(\alpha) = \frac{\lambda}{\pi} \exp(-\lambda |\alpha|^2)$$





(first experiment: Furusawa et al. Science 1998)

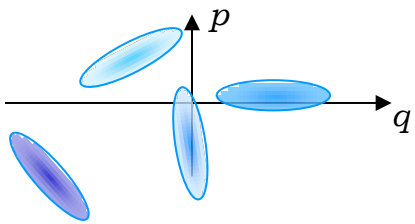
Gaussian benchmarks: special cases



Input: **squeezed states** (unknown s; $\alpha, \theta = 0$)
along one direction, with totally random squeezing degree

Benchmark: $$\bar{F}_C \approx 0.83$$

(Adesso-Chiribella PRL 2008)

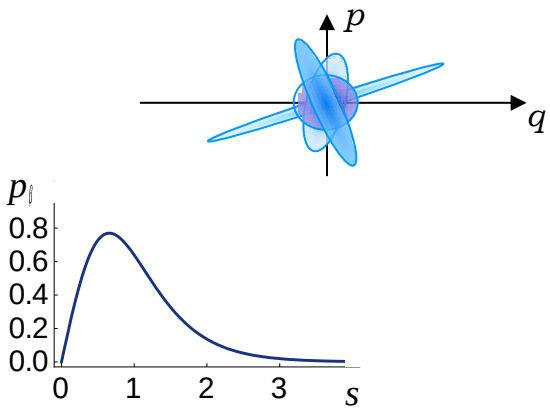


Input: **squeezed states** (known s; unknown α, θ)
with known squeezed degree, random phase and displacement

Benchmark: $$\bar{F}_C = \frac{\operatorname{sech} s}{2}$$

(Owari et al NJP 2008)
(Calsamiglia et al PRA 2009)
(experiment: Jensen et al Nat Phys 2011)

Gaussian benchmarks: this work



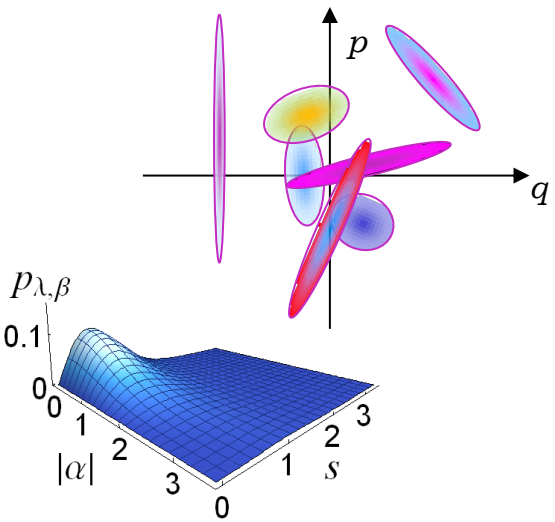
Input: **squeezed states** (unknown s, θ; $\alpha = 0$)

no displacement, random phase, squeezing degree distributed according to a finite-width distribution p_β with inverse width β

Benchmark:

$$\overline{F}_C^{sqz} = \frac{1+\beta}{2+\beta}$$

(Chiribella-Adesso PRL 2014)



Input: **general Gaussian states** (unknown s, θ, α)

with random phase, displacement and squeezing degree distributed according to a finite-width distribution $p_{\lambda,\beta}$ with inverse widths λ and β

Benchmark:

$$\overline{F}_C^{gen} = \left(\frac{1+\lambda}{2+\lambda}\right)\left(\frac{1+\beta}{2+\beta}\right)$$

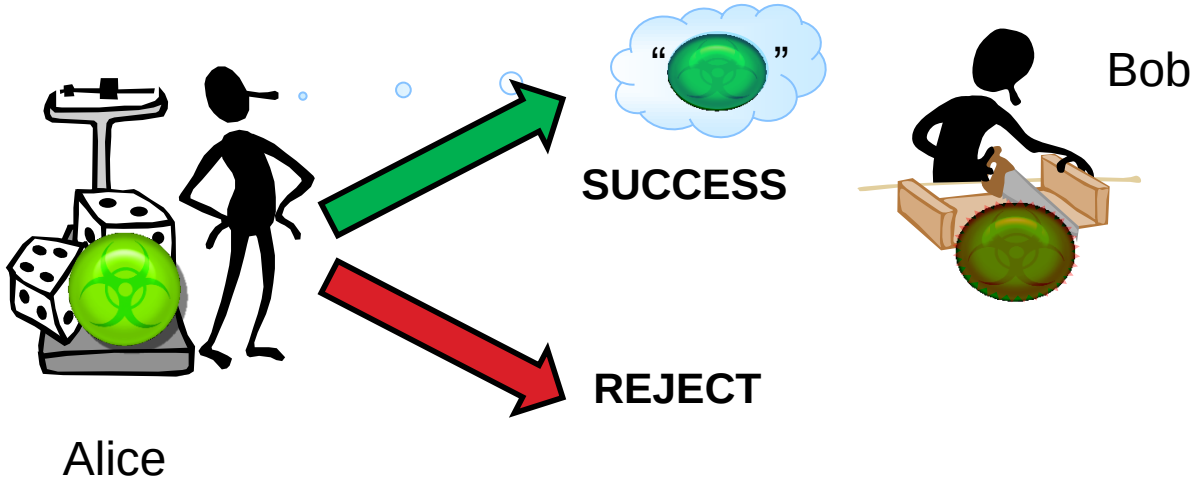
(Chiribella-Adesso PRL 2014)

Technicalities

The benchmarks are generally **probabilistic**

- Alice measures the input $|\psi_x\rangle$ by the POVM $\{\hat{\Pi}_y\}$, Bob reprepares $\hat{\rho}_y$
- They discard runs where the outcomes y are outside a "success" set

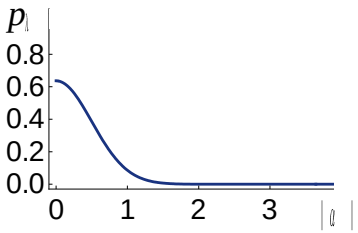
$$\bar{\mathcal{F}}_c = \sum_{x\in X}\sum_{y\in Y_{suc}} p(x|suc)\frac{\langle\psi_x|\hat{\Pi}_y|\psi_x\rangle}{\sum_{y'\in Y_{suc}}\langle\psi_x|\hat{\Pi}_{y'}|\psi_x\rangle}\langle\psi_x|\hat{\rho}_y|\psi_x\rangle$$



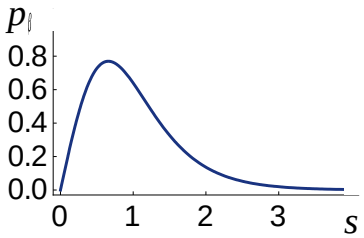
Technicalities

- The proof uses group theory arguments. We derived a formalism to calculate benchmarks for **generalised** (Perelomov) **coherent states**
- The input distributions are generally non-Gaussian, but they originate from the overlap of the considered states with the vacuum, $p \sim |\langle 0|\psi\rangle|$

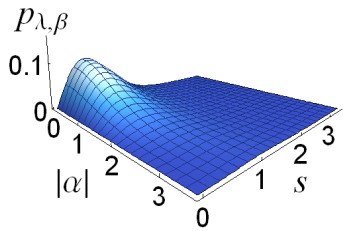
$$p_\lambda^C(\alpha) = \frac{\lambda}{\pi} e^{-\lambda|\alpha|^2}$$



$$p_\beta^S(\xi) = \frac{p_\beta(s)}{2\pi}, \quad \text{with} \quad p_\beta(s) = \frac{\beta \sinh s}{(\cosh s)^{\beta+1}}$$

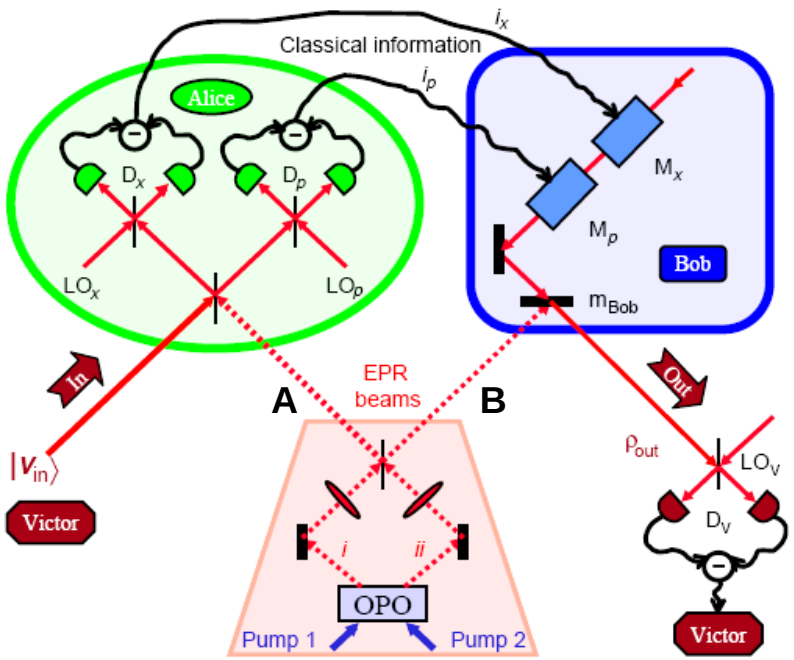


$$p_{\lambda,\beta}^G(\alpha, s, \theta) = \frac{\lambda\beta}{2\pi^2} \frac{e^{-\lambda|\alpha|^2 + \lambda \mathrm{Re}(e^{-i\theta}\alpha^2)\tanh s} \sinh s}{(\cosh s)^{\beta+2}}$$



Beating the benchmarks?

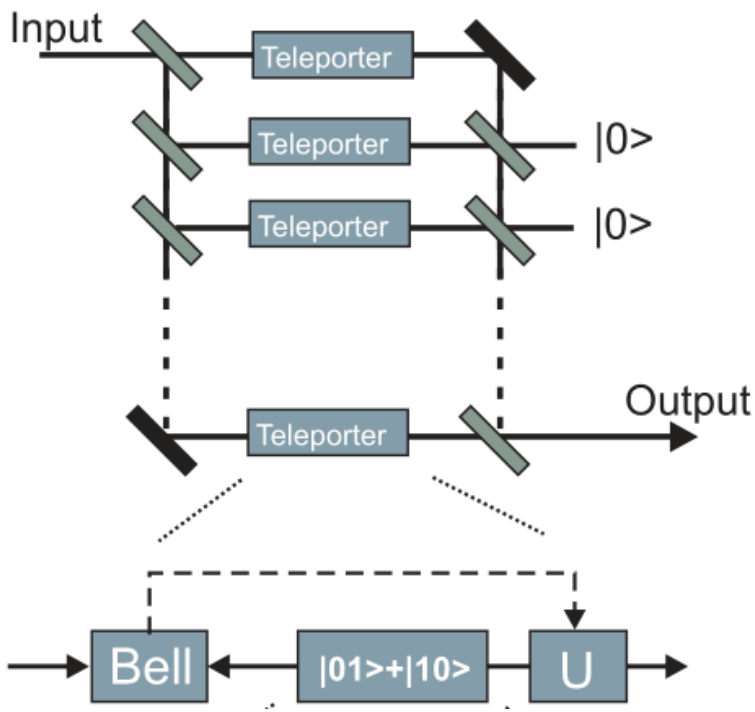
- No experiment yet has teleported arbitrary Gaussian states sampled from a finite-width distribution



- **Continuous variable teleportation protocol** *(Braunstein-Kimble PRL 1998)*
- Shared entangled resource: a two-mode squeezed state with squeezing *r*
- Alice performs a double homodyne, Bob displaces his mode according to the communicated results
- The protocol is deterministic

Beating the benchmarks?

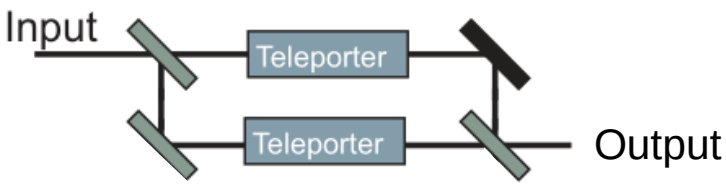
- No experiment yet has teleported arbitrary Gaussian states sampled from a finite-width distribution

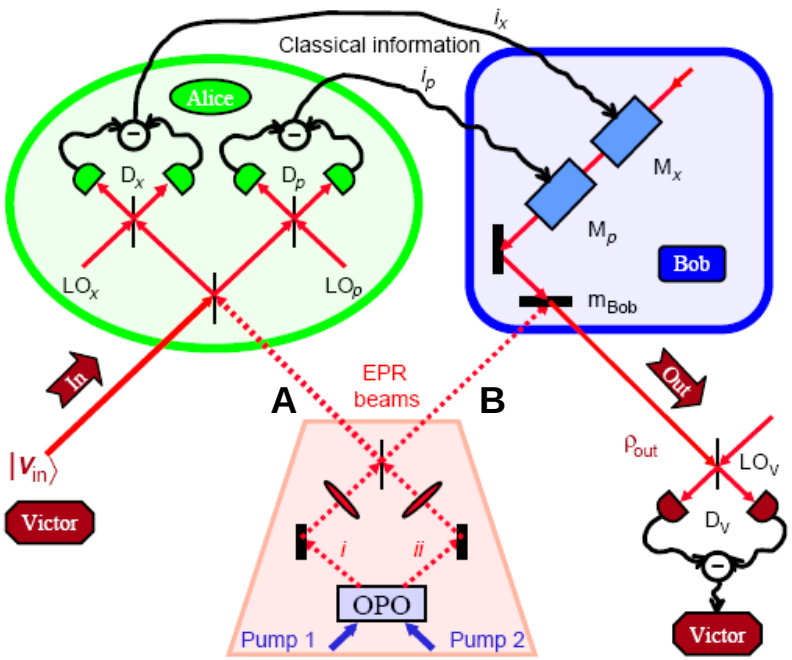


- **Hybrid teleportation protocol** *(Andersen-Ralph PRL 2013)*
- Shared entangled resource: a product of *N* two-qubit maximally entangled states
- Perform *N* discrete-variable teleportation in parallel on a splitted input, then recombine the output at the end
- This protocol is probabilistic

Comparison

- Hybrid protocol with $N = 2$ qubit channels vs
- Braunstein-Kimble with $r = \mathrm{arcsinh}(1)$ [i.e. $\sim$3.8 dB]
- In both cases the shared resources have equal entanglement and mean energy





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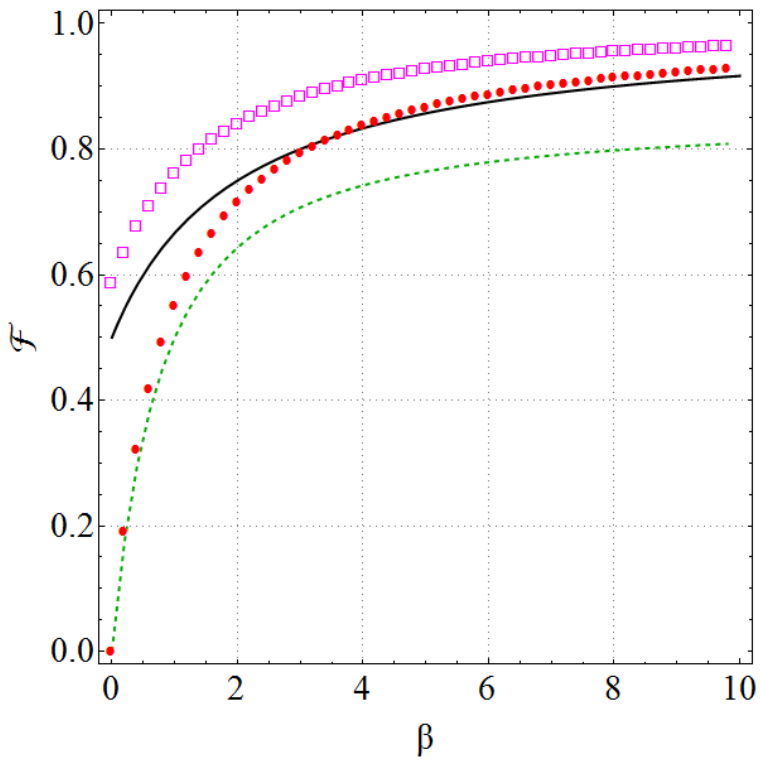


- **Teleportartion of undisplaced squeezed states distributed according to p_β**

Hybrid Andersen-Ralph
B-K with optimised gain
B-K with unit gain
Benchmark

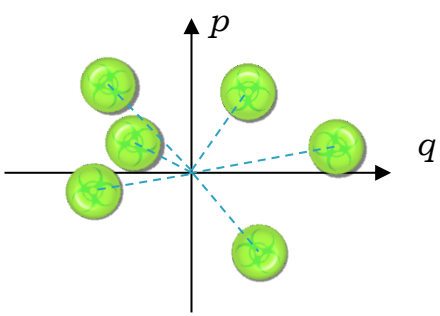
Similar results for other cases…

(I. Kogias, S. Ragy, G.A. PRA 89 (2014)

Summary

G. Chiribella & G.A. Phys. Rev. Lett. 112, 010501 (2014)

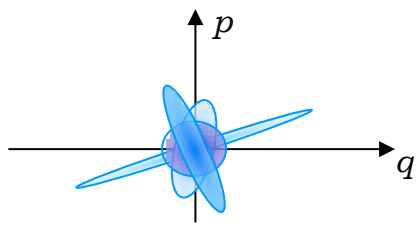
- We obtained **quantum benchmarks** for the teleportation and storage of realistic ensembles of pure single-mode Gaussian states, including the general case
- Summary of values for the classical fidelity threshold:



Displaced, unsqueezed

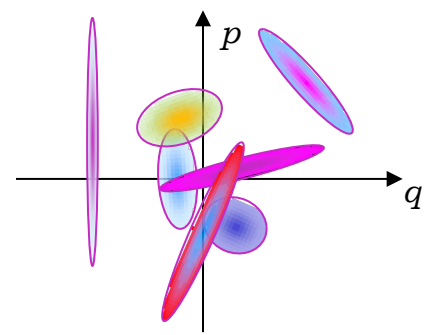
$$\frac{1+\lambda}{2+\lambda}$$

(Hammerer et al 2005)



Undisplaced, squeezed

$$\frac{1+\beta}{2+\beta}$$

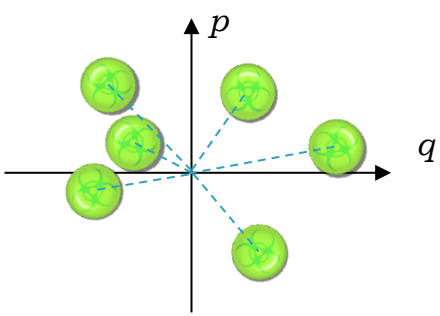


Displaced, squeezed

$$\left(\frac{1+\lambda}{2+\lambda}\right)\left(\frac{1+\beta}{2+\beta}\right)$$

Outlook & Work in progress

- Derive benchmarks for other channels applied to Gaussian states (cloning, amplification, …)
- Derive benchmarks for more general classes of states
- Devise optimal protocols suitable to transmit squeezing
- <u>Demonstrate an experimental violation of the benchmarks!</u>



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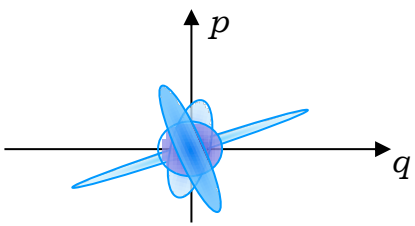
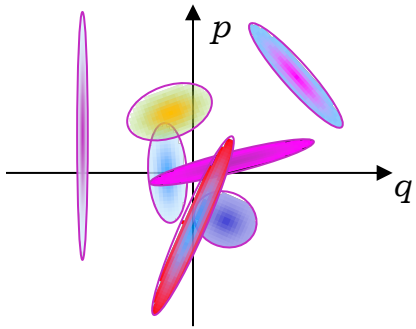


Undisplaced, squeezed

$$\frac{1+\beta}{2+\beta}$$



Displaced, squeezed

$$\left(\frac{1+\lambda}{2+\lambda}\right)\left(\frac{1+\beta}{2+\beta}\right)$$

(Hammerer et al 2005)

THANK YOU

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from The University of Nottingham

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